Testing Homogeneity and Symmetry in a Dynamic Import Allocation Model: an Empirical Study Applied to Switzerland

Thesis

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Roberto Cerratti

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First Adviser

Prof. Dr. Philippe Deschamps

Second Adviser

Prof. Dr. Reiner Wolff

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Abstract

The aim of this Ph.D. thesis is to estimate an import allocation model for Switzerland and test the homogeneity and symmetry hypotheses implied by demand theory. In previous works, the rejection of these hypotheses has often be attributed to the dynamic mis-specification of the models. Some alternative dynamic specifications (error-correction, autoregressive-errors and partial-adjustment) are considered in order to introduce dynamics and check whether or not they produce any differences in the homogeneity and symmetry test results.

The chosen functional form is the AID (Almost Ideal Demand System) model, firstly used by Winters (1984) for explaining international trade flows. Starting from a general dynamic import allocation model (or its equivalent error-correction form) we specify some simpler dynamic specifications like the autoregressive-errors and partial-adjustment models. A static model is also estimated for comparison purposes.

Unconstrained and constrained estimations are computed for each of the dynamic models using maximum likelihood techniques and data of Swiss imported manufactures. The model performances are evaluated according to several criteria (goodness-of-fit, information criteria, residual analysis) and, finally, likelihood ratio statistics are computed to test the economic theory assumptions and the different dynamic specifications.

Introduction

The aim of this Ph.D. thesis is to estimate an import allocation model for Switzerland and to test the homogeneity and symmetry hypotheses implied by demand theory. In the literature this topic has been largely treated in the context of trade allocation (see for example Burgess (1974), Kohli (1978), Winters (1984), Diewert and Morrison (1986, 1988) and more recently van Heeswijk et al. (1993)), but homogeneity and symmetry have often been rejected. The rejection of these hypotheses has commonly be attributed to the dynamic mis-specification. Our goal is to estimate an import allocation model introducing several dynamic specifications and observing what the consequences on the test results are.

The starting point of our work is the static model estimated by Winters (1984). Winters' study was the first to use the AID (Almost Ideal Demand System, Deaton and Muellbauer, 1980a) functional form for explaining international trade flows. The AID model offers a combination of generality and tractability. Its main advantages are that it largely avoids the use of non-linear estimation techniques and it allows to impose homogeneity and symmetry through linear restrictions on fixed parameters. Despite the use of more sophisticated tools like the AID model, Winters rejected both the homogeneity and symmetry assumptions. He identified the neglect of dynamics and the theoretical shortcomings of his model as the causes of his results.

To extend Winters' work, we will define a general dynamic import allocation model of the form $\Phi(L)w_t = \Gamma(L)x_t + u_t$, where w_t is a $n \times 1$ vector of endogenous variables, x_t is $k \times 1$ vector of exogenous variables, and $\Phi(L) = I - \Phi_1 L - \ldots - \Phi_p L^p$ and $\Gamma(L) = \Gamma_0 + \Gamma_1 L + \ldots + \Gamma_q L^q$ are matrix polynomials in the lag operator L (Φ_i is of order $n \times n$, Γ_j of order $n \times k$). Some simpler dynamic forms like the static, autoregressive-errors and partial-adjustment models can be obtained by restricting the polynomials $\Phi(L)$ and $\Gamma(L)$ in the general dynamic model (or in its equivalent error-correction representation). The use of simpler representations is mainly justified by the large number of parameters of the general dynamic model, which could affect the estimates efficiency. The autoregressive-errors and partial-adjustment models could represent a good trade-off between the introduction of dynamics and the number of variables of the model.

For each of the dynamic specifications, we will specify and estimate an unconstrained, homogeneous and symmetric model. The estimation will be carried out using maximum likelihood techniques. Several criteria like goodness-of-fit, information criteria and residual analysis will be used to compare the model performances. Finally, hypothesis testing on the economic theory assumptions and on the dynamic specification will be computed using the likelihood ratio statistic.

The work is organized as follows:

Chapter 1 introduces some basic concepts of production theory, in particular it explains the advantages of choosing the cost minimization approach, which allows to compute conditional factor demands using Shephard's lemma. The country is seen as a firm that chooses its inputs (foreign or domestic) between several suppliers, according to their prices, in order to minimize its costs (under the hypothesis of producing a certain amount of input q).

Chapter 2 presents the import allocation models, in particular the separability assumption and the two-stage budgeting procedure that let us define the import functions we will estimate. We also discuss the choice of the AID (Almost Ideal Demand System) model as a functional form and derive the equations that will compose our demand system. In the second part of the chapter, we introduce dynamics by defining a general dynamic model (or its equivalent error-correction representation) and its static, autoregressive-errors and partial-adjustment restricted forms. Finally, we discuss the implications of the adding-up, homogeneity and symmetry restrictions, in particular how these can be imposed in our models.

Chapter 3 deals with estimation and hypothesis testing. A general maximum likelihood procedure is presented by rewriting the error-correction model as a bilinear model in which the coefficients are estimated iteratively using a quasi-Newton algorithm (the score method). This procedure applies also to the restricted (homogeneous and symmetric – for each dynamic specification) models. The likelihood ratio test statistic presented at the end of the chapter will be used to test these restrictions.

Chapter 4 focuses on data analysis. We firstly describe the data set (taken from the Swiss-Impex database) and explain the importance of stationarity for the equilibrium multipliers estimation. Joint stationarity tests (using the Johansen cointegration test) will be computed for different data specifications: levels, first-differences and seasonal-differences. Once a data specification has been chosen, a procedure based on diagnostic and specification tests will determine the lag length of the matrix polynomials of the general dynamic model.

Chapter 5 presents the estimation results. Equilibrium multipliers, Slutsky matrices and elasticity estimates are discussed for each of the dynamic models and under all economic restrictions. The model performances are evaluated using goodness-of-fit measures, information criteria and residual analysis. Hypothesis testing concludes the chapter. We test, on the one hand, the homogeneity and symmetry restrictions and, on the other hand, the different dynamic specifications.

Chapter 6 summarizes the main results and concludes the work.

Chapter 1

Duality and the Production Theory Approach

1.1 Introduction

In this chapter some basic concepts of production theory are briefly introduced. An overview of the primal (profit maximization) and of the dual (cost minimization) problem is given, focusing the attention on the advantages of the latter one. The main justification of the use of the dual approach is that transformation and cost functions are equivalent representations of an underlying technology, but cost functions allow us to compute the (conditional) factor demands in which we are interested in our analysis.

Assume that the country is a firm that produces output (GDP) from a set of inputs (foreign and domestic) and whose objective is profit maximization or equivalently, under certain hypotheses, cost minimization. Assume also that the economy has L commodities.

A production vector (or production plan) is a vector $y = (y_1, \ldots, y_L) \in \mathbb{R}^L$ that describes the outputs of the L commodities from a production process. An input is denoted by $y_l < 0$, an output by $y_l \geq 0$. The set of all production vectors that constitute feasible plans for the firm (i.e. satisfy technological constraints) is known as the production set (denoted by $Y \subset \mathbb{R}^L$) and it is described by a function $F(\cdot)$, called transformation frontier function, such that $Y = \{y \in \mathbb{R}^L : F(y) \leq 0\}$.

1.2 The Profit Maximization Problem

The market behavior of the firm is often described as the profit maximization problem. Let $p = \Big(p_1 \ldots p_L\Big) \gg 0$ be a vector of prices for the L goods and assume that these prices are independent of the production plans of the firm (i.e. the firms are price-takers). Additionally, assume that the firm's production set Y satisfies the properties of nonemptiness, closedness and free disposal (for details see Mas-Colell, Whinston and Green, 1995) .

A profit maximization firm solves the following problem (called PMP):

$$\begin{aligned} \max_y \quad & p' \cdot y \\ & \text{s.t.} \quad F(y) \leq 0. \end{aligned}$$

The solution to the PMP is the supply correspondence $y(p)$. We obtain the profit function of the firm as $\Pi(p) := \max_y\{p' \cdot y \mid F(y) \leq 0\}$.

In the context of the import demand (and export supply) functions this has been the approach followed for example by Kohli (1978) and Diewert and Morrison (1986). Kohli chose a representation of the technology similar to Samuelson's GNP function and estimated simultaneously import and export functions (together with the demand and supply functions of the domestic factors) that maximize the profit function $\Pi(p)$. From the maximized value of this function, $\Pi(\bar{p})$, and using Hotelling's lemma:

$$y(\bar{p}) = \nabla_p \Pi(\bar{p}),$$

where ∇ is the vector differential operator, he derived demand and supply equations for the variable quantities. Diewert and Morrison followed exactly the same approach but chose an alternative functional form for the profit function ensuring that the curvature conditions imposed by the economic theory were respected.

Profit maximization is not the only possible way to describe the behavior of a firm. An equivalent description of the problem can be given using the dual approach: minimizing costs of inputs under the hypothesis that the firm produces the same amount of output.

1.3 The Cost Minimization Problem

Cost minimization is a necessary condition for profit maximization since there is no way to produce the same amount of output at a lower total input cost. The aim of our work is to estimate import demand functions (or, equivalently, conditional factor demands of foreign inputs) and thus it is natural to focus our attention on the cost side of the problem. Assuming that the country produces a certain amount of output q,[1] our goal is to find the best set of inputs (according to their country of origin and price) that minimize costs.

The cost minimization problem is useful for several reasons. First, its results hold when a firm (or country) is not a price-taker on the output market, as long as input prices are given. Second, with nondecreasing returns to scale, at perfect competition, the PMP does not have a solution, but the results of cost minimization can still be applied. Third, cost minimization is useful to characterize the factor demands of the firm; between these we find the import demand functions in which we are interested.

[1] The primal and dual problem are equivalent if the same amount of output is produced.

Let z be a nonnegative vector of inputs, $f(z)$ the production function, q the amount of output and $p^* \gg 0$ the vector of input prices (note that z and p^* are sub-vectors of, respectively, y, the netput vector, and p, the vector of input-output prices). The cost minimization problem (CMP), in the single-output case, can be defined as:

$$\min_{z \geq 0} \quad p^* \cdot z$$
$$\text{s.t.} \quad f(z) \geq q.$$

The solution of the problem, the optimized value of the CMP, is given by the cost function $c(p^*, q)$ and its corresponding set of input choices, the conditional factor demand correspondence, is $z(p^*, q)$. This can be computed applying Shephard's lemma: if $z(p^*, q)$ consists of a single point, then c is differentiable and $\nabla_{p^*} c(p^*, q) = z(p^*, q)$, where ∇ denotes the vector differential operator.

It has to be noted that the cost function c has several useful properties: it is homogeneous of degree one, concave in p^* and nondecreasing in q.

In fact, the cost function is an alternative characterization (to the transformation or the profit functions) of technology, particularly useful when the production set is of the nondecreasing or the constant returns type. With nondecreasing returns to scale, the solutions of the CMP, which keeps the level of output fixed, are better behaved than the profit function and supply correspondence of the PMP (with nondecreasing returns to scale, the profit function can take only the values 0 and ∞, see, for example, Mas-Colell et al., 1995). With constant returns to scale, the supply correspondence $y(\cdot)$ is not single-valued at any price vector allowing for nonzero production, and thus Hotelling's lemma becomes inapplicable. Yet, the conditional input demand $z(p^*, q)$ may nevertheless be single-valued, allowing the use of Shephard's lemma.

Burgess (1974) estimated import demand functions following the cost minimization approach and using the translog cost function as functional form.

We briefly presented the theoretical basis here that we will use in the next chapters and that justify our approach. In practice, we assume that the country is a firm that chooses its inputs, particularly foreign inputs, between several suppliers (other countries) and according to their prices, in order to minimize its costs and under the hypothesis of the production of a certain amount of output q.

In the next chapters, we will first specify a functional form for the cost function (and various alternative dynamic specifications), set and solve the CMP and from the optimized value of the cost function, compute the factor (or inputs) demands.

Chapter 2

Import Allocation Models

Allocation models refer to a class of systems of equations that purport to explain how some aggregate such as total producer's expenditure is distributed among its component categories in, say, a demand system (Bewley, 1986). The main characteristic of this class of models is that they are subject to a basic identity: the sum of the components equals the aggregate (or in budget share form, the sum of the shares has to be unity).

Clearly, under this class the models allocating a country's imports among suppliers can also be classified. In the family of trade models, whose aim is to explain and predict the effects of relative price changes, this is what we call the trade allocation approach. The literature counts various trade allocation models, for example Burgess (1974), Kohli (1978), Winters (1984) or Diewert and Morrison (1986 and 1988). Between these, Winters proposed a general static model built on the basis of the consumer demand analysis and the AID model introduced by Deaton and Muellbauer (1980a). His model is the starting point of our work.

In this chapter, we will present the import allocation framework, choose a functional form for the cost function and then discuss different dynamic specifications.

2.1 The Framework

We assume that the factors import allocation follows a two-stage budgeting procedure in which the total conditional foreign factor demand (total imports demand) of some good g is explained by a function of the form:

$$z^f = z^f(P^f, P^d, y)$$

and then allocated among suppliers (countries) by equations such as:

$$z_i^f = z_i^f(z^f, p_1^f, \ldots, p_i^f, \ldots, p_n^f)$$

where z^f is the total conditional foreign factor demand of the concerned good (total imports demand of good g), z_i^f is the conditional factor demand from country i (imports demand from country i), p_i^f are factor (import) prices, P^f and P^d are indexes of foreign and domestic factor prices and y is the gross domestic product.

The allocation procedure is characterized by two stages: at the first one, total imports demand depend on the price indexes P^f and P^d and not on the individual prices p_i^f (or p_i^d); at the second stage, the allocation between countries is independent of the determinants of total imports except via total imports themselves. At each stage then, only information appropriate to that stage is required but the result of two-stage budgeting has to be identical to what would occur if the allocation was made in one step with complete information. This is possible by assuming (weak) separability between domestic and foreign inputs at the first stage of the allocation procedure.

The correspondence between weak separability and two-stage budgeting and the existence of sub-cost functions allows to define the group (foreign and domestic) and the "individual" (countries) cost functions in the same way. This implies that the first-stage of the allocation problem can be solved by minimizing total cost given some domestic and import price indexes, and the second stage can be solved by minimizing the import costs subject to total imports z^f and the countries price indexes p_i^f.

It has to be highlighted that weak separability has several implications for the substitutability of goods: it places severe restrictions on the degree of substitutability between inputs in different groups (the substitution effect between goods in different groups is determined by intra-group income effects but only inter-group price effects) and it implies also that the marginal rate of substitution between inputs from the same group is independent of the quantities consumed of inputs in other groups. We will only briefly present here the separability assumptions we require in order to apply the two-stage budgeting procedure to an import allocation problem. For a detailed discussion of these assumptions, see for example Blackorby et al. (1978).

2.1.1 Domestic and Foreign Inputs: Inter-group Allocation

Let divide the vector of inputs z into two subvectors (or groups):

$$z = \begin{pmatrix} z^d & z^f \end{pmatrix}$$

where z^d are domestic inputs and z^f are foreign inputs (similarly, the input prices vector p can be subdivided as $p = \begin{pmatrix} p^d & p^f \end{pmatrix}$). Separability implies sub-group conditional demand functions of the form:

$$z_i^d = z_i^d(p^d, z^d)$$
$$z_i^f = z_i^f(p^f, z^f)$$

which means that we can treat the domestic and the foreign allocation problems separately once this first stage is completed.

2.1.2 Intra-group Allocation: The Import Functions

Assume that the "producer" has already allocated his expenditure between domestic and foreign inputs. We are now interested in how imports (foreign inputs) are allocated between countries (suppliers).

Let denote by $i = 1, \ldots, n$ the countries from which the inputs are imported. Because of separability at the first stage, the conditional factor demand for country i can be expressed as a function of total expenditure on imports (z^f or M) and import prices:

$$z_i^f = z_i^f(p^f, z^f)$$

where $p^f = \begin{pmatrix} p_1^f & \ldots & p_n^f \end{pmatrix}$ denotes the vector of the country's import prices. These, called import functions, are the functions we want to estimate.

In addition to the separability assumption, it is important to choose a flexible functional form that is compatible with the two-stage budgeting procedure. This is the exact aim of the next section.

2.2 The Choice of a Functional Form: The AID Cost Function

The choice of a functional form is a crucial issue of our work. Even if our study has been justified by production theory, what we finally want to estimate is a system of demand functions. This validates the use of tools developed in consumer theory analysis, like the AID system introduced by Deaton and Muellbauer (1980a). Between the advantages of this model, we cite the fact that it gives an arbitrary first-order approximation to any demand system. It is simple to estimate, largely avoiding the need for non-linear estimation and it can be used to test the homogeneity and symmetry constraints through linear restrictions on fixed parameters (Deaton and Muellbauer, 1980a).

We assume then, that the following AID (almost ideal demand system) cost function provides an exact description of the minimum cost of producing output q given the vector of factor prices p:

$$\log c(p,q) = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j + q\beta_0 \prod_k p_k^{\beta_k} \qquad (2.2.1)$$

where p_k (or p_j) is the import price of country k (or j) with $k, j = 1, \ldots, n$ and α_0, α_k and γ^*_{kj} as well as β_0 and β_k are parameters.

Production theory requires that the cost function satisfies several restrictions, in particular homogeneity of degree one in input prices. In order to respect homogeneity, we need that $\sum_{k=1}^n \alpha_k = 1, \sum_{j=1}^n \gamma^*_{kj} = \sum_{k=1}^n \gamma^*_{kj} = \sum_{k=1}^n \beta_k = 0$.

From the cost function, we can derive the conditional demand functions using Shephard's lemma. Logarithmic differentiation of (2.2.1) gives the budget share as a function of input prices and output:[1]

$$\frac{\partial \ \log c(p,q)}{\partial \log p_i} = w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i q \beta_0 \prod p_k^{\beta_k} \qquad (2.2.2)$$

where $\gamma_{ij} = \frac{1}{2}(\gamma^*_{ji} + \gamma^*_{ij})$.

Now, define x as the total expenditure on inputs corresponding to the optimal output. It is clear that this also corresponds to the optimal value of the cost function ($x \equiv c(p,q)$). We can thus express q as a function of input prices and total expenditure

[1] It is a fundamental property of the cost function that its price derivatives are the quantities demanded: $\frac{\partial c(p,q)}{\partial p_i} = q_i$. Multiplying both sides by $p_i/c(p,q)$ we find: $\frac{\partial \log c(p,q)}{\partial \log p_i} = \frac{p_i q_i}{c(p,q)} = w_i$, since $\frac{\partial c(p,q)}{\partial p_i} \frac{p_i}{c(p,q)} = \frac{\partial \log c(p,q)}{\partial \log p_i}$ and, with the optimal output, the cost function corresponds to total expenditure.

using equation (2.2.1):

$$\begin{aligned}
\log \underbrace{c(p,q)}_{x} &= \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j + q\beta_0 \prod_k p_k^{\beta_k} \\
\Longleftrightarrow q &= \frac{1}{\beta_0 \prod_k p_k^{\beta_k}} [\log x - (\alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j)] \\
&= \frac{1}{\beta_0 \prod_k p_k^{\beta_k}} \log \frac{x}{P}
\end{aligned}$$

where P is a price index defined by $\log P = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j$. If we substitute this into equation (2.2.2) we obtain the AID conditional demand functions in budget share form:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log (x/P). \tag{2.2.3}$$

Until this point, no distinction has been made regarding the nature of the expenditure x. In Section 2.1, we assumed separability between domestic and foreign inputs or equivalently between domestic and import expenditure. It follows that the cost function associated to the foreign inputs allocation problem depends on import prices and total expenditure on imports. The associated conditional demand function in budget share form can then be formulated as:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p^f_j + \beta_i \log (M/P^f) \tag{2.2.4}$$

where p^f_j is the input price of country j, M represents total imports and P^f is an import price index defined by $\log P^f = \alpha_0 + \sum_k \alpha_k \log p^f_k + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p^f_k \log p^f_j$. Deaton and Muellbauer (1980a) highlighted the fact that this price index is non-linear and they suggested to use an approximation whose weights are independent of the parameters. They proposed Stone's index:

$$\log P^{f*} = \sum_k w_k \log p^f_k. \tag{2.2.5}$$

If $P^f \simeq \phi P^{f*}$, then (2.2.4) can be estimated as:

$$w_i = \underbrace{(\alpha_i - \beta_i \log \phi)}_{\alpha_i^*} + \sum_j \gamma_{ij} \log p_j^f + \beta_i \log (M/P^{f*}). \tag{2.2.6}$$

The restrictions on the cost function (in order to make it linearly homogeneous) and the definition of γ_{ij} imply restrictions on the parameters of the AID demand functions. Provided that:

$$\sum_{i=1}^{n} \alpha_i = 1, \quad \sum_{i=1}^{n} \gamma_{ij} = 0, \quad \sum_{i=1}^{n} \beta_i = 0, \tag{2.2.7}$$

$$\sum_{j=1}^{n} \gamma_{ij} = 0, \quad i = 1, \ldots, n \tag{2.2.8}$$

$$\gamma_{ij} = \gamma_{ji}, \quad i, j = 1, \ldots, n \tag{2.2.9}$$

(or in the approximated system $\sum_{i=1}^{n} \alpha_i^* = 1$) hold, the AID demand functions described above represent a system of demand functions which add up to total expenditure (or unity in budget shares form), are homogeneous of degree 0 in prices and total expenditure taken together and satisfy Slutsky symmetry.

The usefulness and the opportunity of these restrictions will be discussed further in this work. In consumer demand analysis, homogeneity and symmetry have often been rejected in empirical studies. However, as Deaton and Muellbauer noticed, the rejection may be the result of an inappropriate dynamic specification because the imposition of homogeneity generates positive serial correlation in the residuals. In the next section we will discuss several dynamic and stochastic specifications whose aim is to improve the model performance.

2.3 Extension of the Model: Dynamic Structure and Stochastic Specification

This topic has been widely investigated in consumer demand analysis and it applies the same way to the production theory approach (i.e. the estimation of conditional, to a fixed level of output, demand functions). The literature on demand system equations abounds with examples of empirical studies in which restrictions from economic theory are rejected. Anderson and Blundell (1982, 1983, 1984) identify the root cause of the problem in the econometric approach, in particular in the missing attention to the dynamic structure of the models. Dynamic specification has to be considered because of the existence of adjustment and information costs necessary for an instantaneous equilibrium decision.[2] The autoregressive-errors specification, for example, may be a solution to this problem, but whilst it is a convenient simplification, it seems that a more general unrestricted dynamic formulation is more appropriate.

Let's start by defining a general dynamic model:

$$\Phi(L)w_t = \Gamma(L)x_t + u_t \tag{2.3.1}$$

where

$$w_t = \begin{pmatrix} w_{1t} & w_{2t} & \dots & w_{nt} \end{pmatrix}'$$

is a $n \times 1$ vector of budget shares of the total imports expenditure on a specific good (or group of goods) from n countries at time t,

$$x_t = \begin{pmatrix} 1 & \log p_{1t} & \dots & \log p_{nt} & \log M_t/P_t \end{pmatrix}'$$

[2]Technical progress could also be considered as a factor justifying the presence of dynamics in the model. It could be represented, for example, by a deterministic time trend. However, we decided not to include technical progress because, as it will be shown later in this work, such a trend seems not to be significant.

is a $(n+2)\times 1$ vector of explanatory variables at time t (a constant term, the import prices and the rate between total imports M_t and an import price index P_t), u_t is an independent identically distributed random disturbance vector, and where:

$$\Phi(L) = I - \Phi_1 L - \Phi_2 L^2 - \ldots - \Phi_p L^p$$

$$\Gamma(L) = \Gamma_0 + \Gamma_1 L + \Gamma_2 L^2 + \ldots + \Gamma_q L^q$$

are matrix polynomials in the lag operator L, with Φ_j and Γ_k being of respective dimension $n \times n$ and $n \times (n+2)$ (with $j = 1, \ldots, p$ and $k = 0, \ldots, q$).

This is the most general dynamic formulation. From this formulation, it is simple to specify alternative dynamic structures by imposing restrictions on the matrix polynomials $\Phi(L)$ and $\Gamma(L)$. But there exists also another, unrestricted, specification of the general model: the error-correction model (ECM).

2.3.1 The Error-correction Model

The general dynamic model can be reparameterized in order to retrieve the long-run structure or an observationally equivalent set of equations. Using the alternative specification of Hamilton (1994), a matrix polynomial $A(L)$ can be expressed as:

Lemma: $A(L) = A(1)L + A^*(L)(1-L)$

with $A^*(L) = \sum_{j=0}^{n-1} A_j^* L^j$

where $A_0^* = A_0$ and $A_j^* = -\sum_{s=j+1}^{n} A_s$ for $j = 1, \ldots, n-1$.

Our two polynomials $\Phi(L)$ and $\Gamma(L)$ can be reparameterized in the same way and the dynamic general model (2.3.1) can be formulated as:

$$\Phi(1)w_{t-1} + \Phi^*(L)\triangle w_t = \Gamma(1)x_{t-1} + \Gamma^*(L)\triangle x_t + u_t \qquad (2.3.2)$$

with $\triangle$ denoting the first-difference operator ($\triangle z_t = z_t - z_{t-1}$).

By further manipulation and assuming that $\Phi(L)$ is invertible,

$$\Phi^*(L)\triangle w_t = -\Phi(1)[w_{t-1} - \Pi x_{t-1}] + \Gamma^*(L)\triangle x_t + u_t \tag{2.3.3}$$

where $\Pi := \Phi^{-1}(1)\Gamma(1)$ is the long-run response and $-\Phi(1)[w_{t-1} - \Pi x_{t-1}]$ is the error-correction term.

Example: For $p = q = 1$, $\Phi(L) = I - \Phi_1 L$ and $\Gamma(L) = \Gamma_0 + \Gamma_1 L$ can be reparameterized as:

$$\Phi(L) = \Phi(1)L + \Phi^*(L)(1 - L)$$
$$\Gamma(L) = \Gamma(1)L + \Gamma^*(L)(1 - L)$$

with $\Phi^*(L) = I$ and $\Gamma^*(L) = \Gamma_0$. The error-correction model will be:

$$\Phi(1)w_{t-1} + \triangle w_t = \Gamma(1)x_{t-1} + \Gamma_0\triangle x_t + u_t$$
$$\triangle w_t = -\Phi(1)\left[w_{t-1} - \Pi x_{t-1}\right] + \Gamma_0\triangle x_t + u_t$$

with $\Pi = \Phi^{-1}(1)\Gamma(1)$.

The advantage of the error-correction formulation is that attention is focused upon the long-run structure in the term $(w_{t-1} - \Pi x_{t-1})$, and this is convenient when imposing suitable restrictions such as homogeneity and symmetry. A similar approach has also been used, among others, by Anderson and Blundell (1982, 1984), Assarsson (1996) and Deschamps (1998). On the other hand, the main disadvantage of the error-correction model is the large number of parameters to be estimated. The data available are often limited and the consequence is an insufficient number of degrees of freedom that yields to imprecise estimators and test statistics. This is the reason that justifies the choice of more restricted dynamic specifications like, for example, the autoregressive-errors or partial-adjustment models.

Note that, as pointed out by Davidson et al. (1978), when x_t is not covariance-stationary (i.e. contains unit roots) equation (2.3.1) can be easily reformulated in terms of first-differences:

$$\Phi(L)\triangle w_t = \Gamma(L)\triangle x_t + \varepsilon_t \tag{2.3.4}$$

and consequently, in the error-correction form, only stationary variables appear:

$$\Phi^*(L)\triangle^2 w_t = -\Phi(1)[\triangle w_{t-1} - \Pi\triangle x_{t-1}] + \Gamma^*(L)\triangle^2 x_t + \varepsilon_t$$

again with $\Pi = \Phi(1)^{-1}\Gamma(1)$ and where $\triangle^2$ is the second difference operator ($\triangle^2 w_t = \triangle w_t - \triangle w_{t-1}$). In this formulation, no long term structure appears, but a short term relationship between the first-differences.

In the next sections we will specify three particular forms of the error-correction model: the static, the autoregressive-errors and the partial-adjustment models. All of them can be computed by imposing restrictions on the matrix polynomials $\Phi(L)$ and $\Gamma(L)$.

2.3.2 The Static Model

The static model represents the long-run solution of the dynamic system. It is clear that, in levels, such a formulation has a signification only if the variables are stationary and there exists an equilibrium relationship between them.

This model can be retrieved by imposing the following restrictions on $\Phi(L)$ and

$\Gamma(L)$:

$$\text{(i)} \quad \Phi(L) = I, \quad \text{or equivalently} \quad \Phi_j = 0 \quad \text{for} \quad j > 0 \tag{2.3.5}$$

$$\text{(ii)} \quad \Gamma(L) = \Gamma_0, \quad \text{or equivalently} \quad \Gamma_k = 0 \quad \text{for} \quad k > 0. \tag{2.3.6}$$

Under these assumptions, the error-correction model (2.3.3) reduces to:

$$\begin{aligned} \triangle w_t &= -[w_{t-1} - \Pi x_{t-1}] + \Gamma_0 \triangle x_t + u_t \quad \text{or} \\ w_t &= \Pi x_t + u_t \end{aligned} \tag{2.3.7}$$

since $\Pi = \Gamma_0$ (because of (i) and (ii)), which is exactly the long-run relationship in the error-correction representation. For equation i,

$$w_{it} = \alpha_i + \sum_j \gamma_{ij} \log p_{jt} + \beta_i \log \frac{M_t}{P_t} + u_{it}$$

The model can also be reformulated in first-differences in the case when w_t or x_t contain unit roots:

$$\triangle w_t = \Pi \triangle x_t + \varepsilon_t \tag{2.3.8}$$

where ε_t are independent and identically distributed error terms. The first-differences model has a different economic interpretation and represents short-term dynamics instead of long-term ones.

The static model (in levels) is the one used, for example, by Winters (1984) in the context of import demand functions. But Winters had considerable problems over plausibility (because of the sign of the own-price effects or the sign of the eigenvalues of the Slutsky matrix, etc.) and had to reject the economic theory hypotheses. He mentioned inadequacies of data and neglect of dynamics as the probable sources of his results. The static model is, then, probably too restricted. In the next sections we will present two alternative, and less restricted, dynamic specifications: the autoregressive-errors and the partial-adjustment models.

2.3.3 The Autoregressive-errors Model

This approach was first used by Berndt and Savin (1975) and Lau (1978). They pointed out the consequences of the adding-up condition for the specification of dynamic error processes in allocation systems. The approach has also been largely investigated in Bewley (1986), Deschamps (1993, 1996) and in the specific domain of the import demand functions by van Heeswijk, De Boer and Harkema (1993).

The autoregressive-errors model can be obtained from the general dynamic model (2.3.1) (or the error-correction model (2.3.3)) by imposing a common factor restriction on $\Gamma(L)$. If we assume that:

$$\Gamma(L) = \Phi(L)\Gamma_0 \tag{2.3.9}$$

which implies that $\Gamma_k = -\Phi_k\Gamma_0$ for $k = 1, \ldots, q$ and $\Gamma_k = 0$ for $k > q$, the general dynamic model becomes:

$$\Phi(L)w_t = \Phi(L)\Gamma_0 x_t + u_t.$$

Assuming also that $\Phi(L)$ is invertible:

$$w_t = \Pi x_t + v_t \tag{2.3.10}$$

with:

$$v_t = \Phi^{-1}(L)u_t = \Phi_1 v_{t-1} + \ldots + \Phi_p v_{t-p} + u_t$$

following an $AR(p)$ process and $\Pi = \Phi^{-1}(1)\Gamma(1) = \Phi^{-1}(1)\Phi(1)\Gamma_0 = \Gamma_0$ (i.e. the long-run response is equal to the impact effect). The error-correction model (2.3.3) can be written as:

$$\Phi^*(L)\triangle w_t = -\Phi(1)[w_{t-1} - \Gamma_0 x_{t-1}] + \Phi^*(L)\Gamma_0\triangle x_t + u_t \tag{2.3.11}$$

since $\Pi = \Gamma_0$ and $\Gamma^*(L) = \Phi^*(L)\Gamma_0$.

Example: If we assume for simplicity that $p = q = 1$, the common factor restrictions (2.3.9) imply that $\Gamma_1 = -\Phi_1\Gamma_0$ and $\Gamma_k = 0$ for $k > 1$. The error-correction model (2.3.3) can be written as:

$$\triangle w_t = -\Phi(1)[w_{t-1} - \Pi x_{t-1}] + \Pi\triangle x_t + u_t$$

because $\Pi = \Gamma_0$ and after some manipulation:

$$\begin{aligned}
\triangle w_t + (I - \Phi_1)w_{t-1} &= (I - \Phi_1)\Pi x_{t-1} + \Pi\triangle x_t + u_t \\
(I - \Phi_1 L)w_t &= (I - \Phi_1 L)\Pi x_t + u_t \\
w_t &= \Pi x_t + v_t \qquad (2.3.12)
\end{aligned}$$

with $v_t = (I - \Phi_1 L)^{-1}u_t = \Phi_1 v_{t-1} + u_t$ following an $AR(1)$ process. For equation i,

$$w_{it} = \alpha_i + \sum_j \gamma_{ij} \log p_{jt} + \beta_i \log \frac{M_t}{P_t} + v_{it}$$

where $v_{it} = \Phi_1^i v_{t-1} + u_t = \sum_{j=1}^n \Phi_{1,ij} v_{j,t-1} + u_t$ (with Φ_1^i denoting the i-th row of Φ_1 and $\Phi_{1,ij}$ denoting the i-th row j-th colum element of Φ_1).

If some elements of x_t, or w_t, contain unit roots, the model in levels can still be estimated, but the common statistic tests and distributions are no longer valid. A way to avoid this problem is to write the model in first-differences as:

$$\triangle w_t = \Pi\triangle x_t + \varepsilon_t \qquad (2.3.13)$$

where $\varepsilon_t = v_t - v_{t-1} = \Phi^{-1}(L)\underbrace{\triangle u_t}_{\xi_t} = \Phi_1\varepsilon_{t-1} + \ldots + \Phi_p\varepsilon_{t-p} + \xi_t$. With this formulation and assuming that ξ_t is white noise, the unit roots disappear but the errors still follow an $AR(p)$ process.

Van Heeswijk et al. (1993) chose the autoregressive-errors approach as dynamic specification and tested a dynamic $AR(1)$ model (with different hypotheses on the autoregressive matrix Φ_1) against the static model. They always strongly rejected the static model and could not reject the regularity assumptions. Their results then support the hypothesis that the common rejection of the economic theory restrictions in demand models is, among other things, due to dynamic mis-specification as well as the bias towards rejection of the null hypothesis in asymptotic tests (van Heeswijk et al., 1993).

2.3.4 The Partial-adjustment Model

An alternative and more general dynamic specification is the partial-adjustment model. This approach is justified by the assumption that changes in the budget shares vector w_t are responses to anticipated and unanticipated changes in x_t, in an attempt to maintain a long-run relationship. In particular, adjustment costs may imply a lagged response to a desired change in the composition of the aggregate. The approach, in the field of allocation models, has been firstly introduced by Anderson and Blundell (1982) and largely treated in Bewley (1986).[3]

Again, the model is a particular case of the general dynamic model (2.3.1) (or the error-correction model (2.3.3)) and can be obtained by imposing the following restrictions on the lag structure of $\Gamma(L)$:

$$\Gamma_k = 0 \quad \text{for} \quad k = 1, \ldots, q. \tag{2.3.14}$$

[3]In particular, Bewley assumed that equilibrium is not always maintained because there are cost of adjustment that are balanced against the cost of being out of equilibrium. He showed that generalized adjustment can be justified using a quadratic cost minimization assumption.

The general dynamic model reduces to:

$$
\begin{aligned}
\Phi(L)w_t &= \Gamma_0 x_t + u_t \\
w_t &= \Gamma_0 x_t + \Phi_1 w_{t-1} + \ldots + \Phi_p w_{t-p} + u_t
\end{aligned} \tag{2.3.15}
$$

and the error-correction model becomes:

$$
\Phi^*(L)\triangle w_t = -\Phi(1)\left[w_{t-1} - \Pi x_{t-1}\right] + \Gamma_0 \triangle x_t + u_t \tag{2.3.16}
$$

with $\Pi = \Phi^{-1}(1)\Gamma_0$.

Example: Assuming again that $p = q = 1$, the error-correction model reduces to

$$
\triangle w_t = -\Phi(1)\left[w_{t-1} - \Pi x_{t-1}\right] + \Gamma_0 \triangle x_t + u_t
$$

with $\Pi = \Phi^{-1}(1)\Gamma_0 = (I - \Phi_1)^{-1}\Gamma_0$. After some manipulation, this can also be written as:

$$
\begin{aligned}
\triangle w_t &= -(I - \Phi_1)w_{t-1} + \Phi(1)\Pi x_{t-1} + \Gamma_0 \triangle x_t + u_t \\
\triangle w_t + w_{t-1} &= \Gamma_0 x_{t-1} + \Gamma_0 \triangle x_t + \Phi_1 w_{t-1} + u_t \\
w_t &= \Gamma_0 x_t + \Phi_1 w_{t-1} + u_t
\end{aligned} \tag{2.3.17}
$$

or, for equation i,

$$
w_{it} = \alpha_i + \sum_j \gamma_{ij} \log p_{jt} + \sum_j \phi_{ij} w_{i,t-1} + \beta_i \log \frac{M_t}{P_t} + u_{it}.
$$

If budget shares, prices and real income contain unit roots, we can rewrite the model (2.3.15) in terms of first differences as:

$$
\triangle w_t = \Gamma_0 \triangle x_t + \Phi_1 \triangle w_{t-1} + \ldots + \Phi_p \triangle w_{t-p} + \varepsilon_t
$$

with ε_t following the usual assumptions.

We discussed several approaches here whose aim is to introduce dynamics in the static AID model in order to correct for mis-specification. Other approaches are possible, for a more detailed discussion of dynamic and stochastic specifications see Bewley (1986), Pollak (1992) or Edgerton (1996). We omitted the discussion regarding the identification problems related to the introduction of lagged variables, nor did we mention the estimation particularities of these models. This topics will be discussed in Chapter 3.

2.4 Adding-up, Homogeneity and Symmetry

After having focused our analysis on the choice of a functional form and its dynamic extensions, we will discuss, in this section, three required restrictions (adding-up, homogeneity and symmetry) in order that our system of equations (i.e. the countries conditional factor demand functions) represents a proper demand system. Adding-up is implied by the fact that we are working with an allocation model: budget shares have to sum to unity. Homogeneity and symmetry are implied by economic theory. Symmetry (of the Slutsky matrix), in particular, is a required restriction, in the context of demand analysis, to ensure the regularity of the system (it is implied by the concavity in prices of the Hicksian demand functions, see for example Mas Colell et al., 1995). Another required condition on the Slutsky matrix is its negative semidefiniteness, here, for simplicity, it will only be checked and not imposed.

Besides their theoretical justification, the consequences of imposing these restrictions are to reduce the number of parameters to be estimated. All the models presented above, except the static model, count a large number of parameters while the number of available data is limited. By imposing these restrictions we want, then, to increase the number of degrees of freedom and thus improve the efficiency of our estimations.

2.4.1 Adding-up

A system of equations purporting to explain the distribution of a predetermined aggregate among its components must satisfy the constraint that the sum of the individual components equals the aggregate. The consequence on the model specification is that the components predicted or implied by the model sum exactly to the predetermined aggregate (Bewley, 1986).

Let's consider the general error-correction model (for simplicity we assume that $p = q = 1$),

$$\triangle w_t = -\Phi(1)[w_{t-1} - \Pi x_{t-1}] + \Gamma_0 \triangle x_t + u_t. \tag{2.4.1}$$

It is obvious that the budget shares have to sum to unity, thus:

$$i_n' w_t = 1,$$

(or equivalently $i_n' \triangle w_t = 0$, with $i_n' = (1 \ \dots \ 1)$) which implies that (Anderson and Blundell, 1982):

$$i' \Phi(1) = k i' \tag{2.4.2}$$

$$i' \Pi = \begin{pmatrix} 1 & 0 & \dots & 0 \end{pmatrix} \tag{2.4.3}$$

$$i' \Gamma_0 = \begin{pmatrix} 0 & \dots & 0 \end{pmatrix} \tag{2.4.4}$$

where k is an unknown scalar and $\Pi = \begin{pmatrix} C \mid S \end{pmatrix}$ with:

$$C = \begin{pmatrix} \alpha_1 & \beta_1 \\ \vdots & \vdots \\ \alpha_n & \beta_n \end{pmatrix} \quad \text{and} \quad S = \begin{pmatrix} \gamma_{11} & \dots & \gamma_{1n} \\ \vdots & \ddots & \vdots \\ \gamma_{n1} & \dots & \gamma_{nn} \end{pmatrix}.$$

Adding-up also implies that:

$$i'u_t = 0, \tag{2.4.5}$$

and it follows that the disturbances have a singular distribution because the last row of (2.4.1) is redundant. Assuming that the disturbances covariance matrix has only one zero eigenvalue, by deleting one component of u_t we solve the problem of singularity.[4] So, let w_t^n (or $\triangle w_t^n$), Π^n, Γ_0^n and u_t^n be w_t ($\triangle w_t$), Π, Γ_0 and u_t with the last row deleted, and (Deschamps, 1993):

$$\Phi^n(1) = \begin{pmatrix} I_{n-1} & 0_{(n-1)\times 1} \end{pmatrix} \Phi(1) \begin{pmatrix} I_{n-1} & -i_{n-1} \end{pmatrix}'.$$

The model can be rewritten in its incomplete (and estimable) form as:

$$\triangle w_t^n = -\Phi^n(1)[w_{t-1}^n - \Pi^n x_{t-1}] + \Gamma_0^n \triangle x_t + u_t^n \tag{2.4.6}$$

Because of (2.4.3) and (2.4.4), the coefficients of the last equation can be easily retrieved from the first $n-1$ ones by multiplying Π^n by $-i'_{n-1}$:

$$\begin{aligned} -i'_{n-1}\Pi^n &= \begin{pmatrix} -\sum_{i=1}^{n-1}\alpha_i & -\sum_{i=1}^{n-1}\beta_i & -\sum_{i=1}^{n-1}\gamma_{i1} & \dots & -\sum_{i=1}^{n-1}\gamma_{in} \end{pmatrix} \\ &= \begin{pmatrix} (\alpha_n - 1) & \beta_n & \gamma_{n1} & \dots & \gamma_{nn} \end{pmatrix} \end{aligned}$$

and, similarly, $-i'_{n-1}\Gamma_0^n$ is the last row of Γ_0 (Barten, 1969).

However, this procedure does not apply to $\Phi(1)$ because its columns have to sum to an unknown scalar k. Thus, $\Phi(1)$ is not identifiable, but this is of minor importance for our study.

---

[4] Barten (1969) showed that it is possible to delete one equation from the system without losing any information. The choice of the dropped equation is arbitrary.

2.4.2 Homogeneity

Homogeneity implies that:

$$\sum_{j=1}^{n} \gamma_{ij} = 0$$

for each $i = 1, \ldots, n$, or equivalently that:

$$Si_n = \begin{pmatrix} 0 & \ldots & 0 \end{pmatrix}' \tag{2.4.7}$$

where S is the matrix of price coefficients and i_n a $n \times 1$ column vector whose elements are equal to unity. In order to impose these restrictions it is useful to rewrite the (incomplete) model in terms of the relative prices p_{it}/p_{nt} .[5] Consider:

$$x_t^h = \begin{pmatrix} 1 & \ln \frac{M_t}{P_t} & \ln \frac{p_{1t}}{p_{nt}} & \ldots & \ln \frac{p_{n-1,t}}{p_{nt}} & \ln p_{nt} \end{pmatrix}' \tag{2.4.8}$$

$$\triangle x_t^h = \begin{pmatrix} \triangle \ln \frac{M_t}{P_t} & \triangle \ln \frac{p_{1t}}{p_{nt}} & \ldots & \triangle \ln \frac{p_{n-1,t}}{p_{nt}} & \triangle \ln p_{nt} \end{pmatrix}' \tag{2.4.9}$$

$$w_t^n = \begin{pmatrix} w_{1t} & \ldots & w_{n-1,t} \end{pmatrix}' \tag{2.4.10}$$

$$\Pi^h = \begin{pmatrix} \alpha_1 & \beta_1 & \gamma_{11} & \ldots & \gamma_{1,n-1} & \sum_{j=1}^n \gamma_{1j} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \alpha_{n-1} & \beta_{n-1} & \gamma_{n-1,1} & \ldots & \gamma_{n-1,n-1} & \sum_{j=1}^n \gamma_{n-1,j} \end{pmatrix} \tag{2.4.11}$$

$$\Gamma_0^h = \begin{pmatrix} \beta_1^0 & \gamma_{11}^0 & \ldots & \gamma_{1,n-1}^0 & \sum_{j=1}^n \gamma_{1j}^0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \beta_{n-1}^0 & \gamma_{n-1,1}^0 & \ldots & \gamma_{n-1,n-1}^0 & \sum_{j=1}^n \gamma_{n-1,j}^0 \end{pmatrix}. \tag{2.4.12}$$

The model can be rewritten as:

$$\triangle w_t^n = -\Phi^n(1)[w_{t-1}^n - \Pi^h x_{t-1}^h] + \Gamma_0^h \triangle x_t^h + u_t^n \tag{2.4.13}$$

and homogeneity can be imposed by setting the last column of Π^h equal to the null vector. The coefficients of the last equation can be computed by multiplying Π^h

[5]This can be done by adding and subtracting $\sum_{j=1}^{n-1} \gamma_{ij} \log p_n$ to the right-hand side of each equation i.

by $-i'_{n-1}$ as shown above. The last coefficient, γ_{nn}, can be obtained by subtracting $\sum_{j=1}^{n-1} \gamma_{nj}$ from the last element of $-i'_{n-1}\Pi^h$. Defining now:

$$
\begin{aligned}
x_t^* &= \begin{pmatrix} 1 & \ln \frac{M_t}{P_t} & \ln \frac{p_{1t}}{p_{nt}} & \dots & \ln \frac{p_{n-1,t}}{p_{nt}} \end{pmatrix}' \\
\triangle x_t^* &= \begin{pmatrix} \triangle \ln \frac{M_t}{P_t} & \triangle \ln \frac{p_{1t}}{p_{nt}} & \dots & \triangle \ln \frac{p_{n-1,t}}{p_{nt}} \end{pmatrix}' \\
\Pi^* &= \begin{pmatrix} \alpha_1 & \beta_1 & \gamma_{11} & \dots & \gamma_{1,n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha_{n-1} & \beta_{n-1} & \gamma_{n-1,1} & \dots & \gamma_{n-1,n-1} \end{pmatrix} \\
\Gamma_0^* &= \begin{pmatrix} \beta_1^0 & \gamma_{11}^0 & \dots & \gamma_{1,n-1}^0 \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{n-1}^0 & \gamma_{n-1,1}^0 & \dots & \gamma_{n-1,n-1}^0 \end{pmatrix},
\end{aligned}
$$

as the vectors of variables and coefficient matrices under homogeneity (i.e. by deleting the last row of x_t^h and the last column of Π^h and Γ_0^h), we can rewrite the homogeneous model as:

$$\triangle w_t^n = -\Phi^n(1)[w_{t-1}^n - \Pi^* x_{t-1}^*] + \Gamma_0^* \triangle x_t^* + u_t^n \tag{2.4.14}$$

or by regrouping the coefficients:

$$
\begin{aligned}
\triangle w_t^n + \Phi^n(1) w_{t-1}^n &= \Phi^n(1) \left[\Psi^* \triangle x_t^* + \Pi^* x_{t-1}^*\right] + u_t^n \\
\Phi^n(L) w_t^n &= \Phi^n(1) \begin{pmatrix} \Psi^* & \Pi^* \end{pmatrix} \begin{pmatrix} \triangle x_t^* \\ x_{t-1}^* \end{pmatrix} + u_t^n
\end{aligned}
$$

where $\Psi^* = \Phi^{n^{-1}}(1)\Gamma_0^*$. In vec form:

$$
\begin{aligned}
\text{vec } \Phi^n(L) w_t^n &= \text{vec } \left[\Phi^n(1) \begin{pmatrix} \Psi^* & \Pi^* \end{pmatrix} \begin{pmatrix} \triangle x_t^* \\ x_{t-1}^* \end{pmatrix}\right] + \text{vec } u_t^n \\
\Phi^n(L) w_t^n &= \left[\begin{pmatrix} \triangle x_t^{*\prime} & x_{t-1}^{*\prime} \end{pmatrix} \otimes \Phi^n(1)\right] \begin{pmatrix} \text{vec } \Psi^* \\ \text{vec } \Pi^* \end{pmatrix} + u_t^n \qquad (2.4.15)
\end{aligned}
$$

since vec $(ABC) = (C' \otimes A)$ vec B and $\Phi^n(L) w_t^n$, u_t^n are vectors.

2.4.3 Symmetry

The notation we introduced at the end of the previous section is convenient for imposing the symmetry constraints. Symmetry requires that the $(n-1)\times(n-1)$ block of price coefficients (γ_{ij}) in Π^* is symmetric, i.e. $\gamma_{ij} = \gamma_{ji}$ for $i, j = 1, \ldots, n-1$.

We rewrite the model in a way where only the lower (or upper) triangle of the price coefficient matrix appears. Define:

$$\pi_s = (\alpha_1 \ \ldots \ \alpha_{n-1} \ \beta_1 \ \ldots \ \beta_{n-1} \ \gamma_{11} \ \gamma_{21} \ \gamma_{22} \ \gamma_{31} \ \ldots \ \gamma_{n-1,n-1})' \tag{2.4.16}$$

and S^n be S with the last row and last column deleted. We have to find a selection matrix (or duplication matrix, see Magnus and Neudecker, 1988) D_s such as:

$$\begin{pmatrix} \text{vec } \Psi^* \\ \text{vec } \Pi^* \end{pmatrix} = \underbrace{\begin{pmatrix} I_{n(n-1)} & 0 \\ 0 & D_s \end{pmatrix}}_{D} \underbrace{\begin{pmatrix} \text{vec } \Psi \\ \pi_s \end{pmatrix}}_{\theta_s} = D\theta_s. \tag{2.4.17}$$

Deschamps (1988) defined:

$$D_s = \begin{pmatrix} I_{r(n-1)} & 0 \\ 0 & L \end{pmatrix} \tag{2.4.18}$$

where r is the number of unconstrained columns of Π^* (in our particular case $r = 2$) and L is an $(n-1)^2 \times n(n-1)/2$ matrix such that vec S^n equals L times the stacked lower triangle of S^n.

Under symmetry, and homogeneity, equation (2.4.15) can be rewritten as:

$$\Phi^n(L) w_t^n = \left[\begin{pmatrix} \triangle x_t^{*\prime} & x_{t-1}^{*\prime} \end{pmatrix} \otimes \Phi^n(1) \right] D\theta_s + u_t^n. \tag{2.4.19}$$

This notation is useful because the coefficients to estimate appear directly in the model. The model simplifies if we consider its particular cases presented in the previous section, i.e. the static, autoregressive-errors and partial-adjustment models.

Static Model: By applying the vec operator to the static model (2.3.7), we have that (with $p = q = 1$):

$$w_t^n = (x_t' \otimes I_{n-1}) \ \text{vec}\ \Pi^n + u_t^n. \tag{2.4.20}$$

Using the x_t^* and Π^* defined above, the homogeneous model can be written as follows:

$$w_t^n = (x_t^{*\prime} \otimes I_{n-1}) \ \text{vec}\ \Pi^* + u_t^n \tag{2.4.21}$$

and the symmetric static model as:

$$w_t^n = (x_t^{*\prime} \otimes I_{n-1})\, D_s \pi_s + u_t^n \tag{2.4.22}$$

where π_s is the vector regrouping, in addition to the unconstrained coefficients, the lower (or upper) triangle of the price coefficients and D_s is a duplication matrix.

Autoregressive-errors Model: In the example of Section 2.3.3, we showed that, with $p = q = 1$, the common factor restrictions $\Gamma(L) = \Phi(L)\Gamma_0$ let us rewrite the model as:

$$\begin{aligned} w_t^n &= \Pi^n x_t + v_t^n \quad \text{with} \\ v_t^n &= \Phi_1^n v_{t-1}^n + u_t^n. \end{aligned}$$

Substituting v_t^n in the first equation and since $v_{t-1}^n = w_{t-1}^n - \Pi^n x_{t-1}$, the model can be arranged as follows:

$$\begin{aligned} w_t^n &= \Pi^n x_t + \Phi_1^n (w_{t-1}^n - \Pi^n x_{t-1}) + u_t^n \\ w_t^n - \Phi_1 w_{t-1}^n &= \Pi^n x_t - \Phi_1^n \Pi^n x_{t-1} + u_t^n \\ \Phi^n(L) w_t^n &= \Pi^n x_t - \Phi_1^n \Pi^n x_{t-1} + u_t^n \end{aligned}$$

or in vec form:

$$\Phi^n(L)w_t^n = \left[(x_t' \otimes I_{n-1}) - (x_{t-1}' \otimes \Phi_1^n)\right] \ \text{vec}\,\Pi^n + u_t^n. \qquad (2.4.23)$$

The homogeneous model can be written using relative prices (taking x_t^*) and the related coefficients matrix Π^*:

$$\Phi^n(L)w_t^n = \left[(x_t^{*\prime} \otimes I_{n-1}) - (x_{t-1}^{*\prime} \otimes \Phi_1^n)\right] \ \text{vec}\,\Pi^* + u_t^n. \qquad (2.4.24)$$

The symmetric model is obtained by imposing $\text{vec}\,\Pi^* = D_s\pi_s$:

$$\Phi^n(L)w_t^n = \left[(x_t^{*\prime} \otimes I_{n-1}) - (x_{t-1}^{*\prime} \otimes \Phi_1^n)\right] D_s\pi_s + u_t^n. \qquad (2.4.25)$$

Partial-adjustment Model: The partial-adjustment model is characterized by the restrictions $\Gamma_i = 0$ for $i = 1, \ldots, q$. With $p = q = 1$, it is equal to:

$$\Phi(L)^n w_t = \Phi^n(1)\Pi^n x_t + u_t^n \qquad (2.4.26)$$

or, in vec form:

$$\Phi^n(L)w_t = [x_t' \otimes \Phi^n(1)] \ \text{vec}\,\Pi^n + u_t^n. \qquad (2.4.27)$$

The homogeneous model:

$$\Phi^n(L)w_t = [x_t^{*\prime} \otimes \Phi^n(1)] \ \text{vec}\,\Pi^* + u_t^n. \qquad (2.4.28)$$

is obtained as before by rewriting the model in terms of relative prices and omitting the reference price (and its corresponding column of the coefficients matrix). By imposing $\text{vec}\,\Pi^* = D_s\pi_s$ we get the symmetric model:

$$\Phi^n(L)w_t = [x_t^{*\prime} \otimes \Phi^n(1)]D_s\pi_s + u_t^n. \qquad (2.4.29)$$

In Chapter 3, we will present an estimation and hypothesis testing procedure for the general error-correction model and we will briefly discuss the characteristics of each of its dynamically restricted forms we are interested in. In particular, we will focus on the differences related to the imposition of homogeneity and symmetry and their consequences for estimation.

Chapter 3

Estimation and Hypothesis Testing

3.1 Introduction

In the previous chapter we presented a general dynamic, a static, an autoregressive-errors and a partial-adjustment model in their unrestricted and restricted forms. The obvious next step of our analysis would be to discuss the estimation and hypothesis testing techniques we will use. In Section 3.2, we present a maximum likelihood estimation procedure that will provide consistent, asymptotically unbiased and asymptotically efficient estimators. This procedure allows us to compute unrestricted and restricted (under homogeneity and symmetry) estimators using the results of the previous chapter. Regularity and dynamic specification restrictions will be tested using the likelihood ratio principle.

In what follows, we will consider only the incomplete model (thus with the last equation deleted) [1] and we will present the estimation procedure with the model in levels. If the variables are non-stationary, it will be straightforward to apply the procedure to the first-differences model.

[1] We will skip the "n" superscripts in order to simplify the notation. For the same reason we will assume that $p = q = 1$.

3.2 Restricted and Unrestricted Maximum Likelihood Estimation

3.2.1 Unrestricted ML Estimation

The error-correction model for our (incomplete) allocation system can be written as (with again, for simplicity, $p = q = 1$):

$$\begin{aligned} \triangle w_t &= -\Phi(1)[w_{t-1} - \Pi x_{t-1}] + \Gamma_0 \triangle x_t + u_t && (3.2.1) \\ \triangle w_t + \Phi(1) w_{t-1} &= \Phi(1)[\Pi x_{t-1} + \underbrace{\Phi^{-1}(1)\Gamma_0}_{\Psi} \triangle x_t] + u_t \\ \Phi(L) w_t &= \Phi(1)\left[\Pi x_{t-1} + \Psi \triangle x_t\right] + u_t. && (3.2.2) \end{aligned}$$

with w_t, x_t, $\triangle x_t$, Π and Γ_0 defined as in Section 2.4. Letting $z_t = \Pi x_{t-1} + \Psi \triangle x_t$, equation (3.2.2) becomes:

$$\begin{aligned} \Phi(L) w_t &= \Phi(1) z_t + u_t \\ w_t - \Phi_1 w_{t-1} &= z_t - \Phi_1 z_t + u_t \\ (w_t - z_t) &= \Phi_1 (w_{t-1} - z_t) + u_t && (3.2.3) \end{aligned}$$

or by applying the vec operator:

$$(w_t - z_t) = \left[(w_{t-1} - z_t)' \otimes I_{n-1}\right] \text{ vec } \Phi_1 + u_t. \tag{3.2.4}$$

Another way to reformulate equation (3.2.1) has been presented in the previous chapter where:

$$\begin{aligned} \Phi(L) w_t &= \left[\begin{pmatrix} \triangle x_t' & x_{t-1}' \end{pmatrix} \otimes \Phi(1)\right] \begin{pmatrix} \text{vec } \Psi \\ \text{vec } \Pi \end{pmatrix} + u_t && (3.2.5) \\ &= \left[\begin{pmatrix} \triangle x_t' & x_{t-1}' \end{pmatrix} \otimes I_{n-1} - \begin{pmatrix} \triangle x_t' & x_{t-1}' \end{pmatrix} \otimes \Phi_1\right] \begin{pmatrix} \text{vec } \Psi \\ \text{vec } \Pi \end{pmatrix} + u_t. && (3.2.6) \end{aligned}$$

Equations (3.2.4) and (3.2.5) are two alternative formulations of the error-correction model (3.2.1). Now, this one is nonlinear, with obvious consequences on the estimators and their distributions. However, for given Π and Ψ, equation (3.2.4) is a linear model in $(w_{t-1} - z_t)$; and for a given Φ_1, equation (3.2.5) is a multivariate regression model with coefficient vectors vec Ψ and vec Π. We, thus, have a bi-linear model given respectively Π, Ψ and Φ_1 which can be easier estimated than equation (3.2.1).

Assume now that U is multivariate normal, i.e. vec $U \sim N(0, I_T \otimes \Omega)$ (or $u_t \sim N(0, \Omega)$ for $t = 1, \ldots, T$), where Ω is a positive definite matrix of order $n-1$. The loglikelihood function associated to equation (3.2.3) can be written as:

$$\begin{aligned} L_1(\Phi_1, \Omega \mid \Pi, \Psi) = & -\frac{(n-1)T}{2} \log 2\pi - \frac{T}{2} \log |\Omega| \\ & - \frac{1}{2} \sum_{t=1}^{T} \left[(w_t - z_t) - \Phi_1 (w_{t-1} - z_t) \right]' \Omega^{-1} \left[(w_t - z_t) - \Phi_1 (w_{t-1} - z_t) \right] \end{aligned} \quad (3.2.7)$$

and the ML estimator of Φ_1 is simply the ordinary least square estimator of equation (3.2.3):

$$\widehat{\Phi}_1 = \left[\sum_{t=1}^{T} (w_t - z_t)(w_{t-1} - z_t)' \right] \left[\sum_{t=1}^{T} (w_{t-1} - z_t)(w_{t-1} - z_t)' \right]^{-1}. \quad (3.2.8)$$

On the other hand, the loglikelihood associated to (3.2.5) is:

$$L_2(\Psi, \Pi, \Omega \mid \Phi_1) = -\frac{(n-1)T}{2} \log 2\pi - \frac{T}{2} \log |\Omega| - \frac{1}{2} \sum_{t=1}^{T} \left[u_t' \Omega^{-1} u_t \right] \quad (3.2.9)$$

where:

$$u_t = \Phi(L) w_t - \left[\begin{pmatrix} \triangle x_t' & x_{t-1}' \end{pmatrix} \otimes \Phi(1) \right] \begin{pmatrix} \text{vec } \Psi \\ \text{vec } \Pi \end{pmatrix}$$

but given Φ_1, the ML estimator of vec Ψ and vec Π can be obtained by applying generalized least squares (because of the normality of the error term and the positive definiteness of Ω) to (3.2.5):

$$\begin{pmatrix} \text{vec } \widehat{\Psi} \\ \text{vec } \widehat{\Pi} \end{pmatrix} = \left[\sum_{t=1}^{T} \left[\begin{pmatrix} \triangle x_t' & x_{t-1}' \end{pmatrix} \otimes \widehat{\Phi}(1) \right]' \widehat{\Omega}^{-1} \left[\begin{pmatrix} \triangle x_t' & x_{t-1}' \end{pmatrix} \otimes \widehat{\Phi}(1) \right] \right]^{-1}$$

$$\cdot \sum_{t=1}^{T} \left[\begin{pmatrix} \triangle x_t' & x_{t-1}' \end{pmatrix} \otimes \widehat{\Phi}(1) \right]' \widehat{\Omega}^{-1} \widehat{\Phi}(L) w_t \quad (3.2.10)$$

where:

$$\widehat{\Omega} = \frac{1}{T} \sum_{t=1}^{T} \widehat{u}_t \widehat{u}_t' \quad (3.2.11)$$

$$\widehat{u}_t = \widehat{\Phi}(L) w_t - \left[\begin{pmatrix} \triangle x_t' & x_{t-1}' \end{pmatrix} \otimes \widehat{\Phi}(1) \right] \begin{pmatrix} \text{vec } \widehat{\Psi} \\ \text{vec } \widehat{\Pi} \end{pmatrix} \quad (3.2.12)$$

$$\widehat{\Phi}(L) w_t = (I_{n-1} - \widehat{\Phi}_1 L) w_t \quad (\text{with } p = q = 1)$$

and where $\widehat{\Phi}_1$ is given by equation (3.2.8).

Using this procedure, in order to maximize L_1 and L_2 we only need to solve:

$$\frac{\partial L_1}{\partial \Phi_1} = 0$$
$$\frac{\partial L_2}{\partial \Omega} = 0$$

using a quasi-Newton algorithm (the score method, thus using the information matrix as an approximation to the Hessian), meanwhile estimating Ψ and Π at each iteration of this algorithm.

The information matrix is defined as:

$$R(\varphi) = -E \left[\frac{\partial^2 L}{\partial \varphi \partial \varphi'} \right] = E \left[\frac{\partial L}{\partial \varphi} \frac{\partial L}{\partial \varphi'} \right] \quad (3.2.13)$$

where $\varphi' = \begin{pmatrix} \varphi_1' & \varphi_2' & \varphi_3' \end{pmatrix}$ is the vector of parameters of the model with:

$$\begin{aligned}
\varphi_1' &= \begin{pmatrix} \text{vec}\,'\Psi & \text{vec}\,'\Pi \end{pmatrix} \\
\varphi_2' &= \text{vec}\,'\Phi_1 \\
\varphi_3' &= \text{vec}\,'\Omega.
\end{aligned}$$

The function L is the full loglikelihood function corresponding to (3.2.2) and is defined as:

$$\begin{aligned}
L(\varphi) = \sum_{t=1}^{T} L_t &= \sum_{t=1}^{T} \left[-\frac{n-1}{2} \log 2\pi - \frac{1}{2} \log |\Omega| - \frac{1}{2} u_t' \Omega^{-1} u_t \right] \\
&= -\frac{(n-1)T}{2} \log 2\pi - \frac{T}{2} \log |\Omega| - \frac{1}{2} \sum_{t=1}^{T} u_t' \Omega^{-1} u_t \qquad (3.2.14)
\end{aligned}$$

with $u_t = \Phi(L) w_t - \Phi(1) [\Pi x_{t-1} + \Psi \triangle x_t]$. Assuming that the vectors $\partial L_t / \partial \varphi$ are martingale differences and that a central limit theorem is applicable, it follows that (Hamilton, 1994):

$$\begin{aligned}
\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \frac{\partial L_t}{\partial \varphi} &\underset{L}{\Longrightarrow} N \left(0, \lim \frac{1}{T} \sum_{t=1}^{T} E \left(\frac{\partial L_t}{\partial \varphi} \frac{\partial L_t}{\partial \varphi'} \right) \right) \\
&\underset{L}{\Longrightarrow} N \left(0, \text{plim}\ \frac{1}{T} \sum_{t=1}^{T} R_t(\varphi) \right).
\end{aligned}$$

Maximum likelihood estimation then implies that:

$$\sqrt{T} \left(\widehat{\varphi} - \varphi \right) \underset{L}{\Longrightarrow} N \left(0, \text{plim}\ T R^{-1}(\varphi) \right)$$

with $R(\varphi) = \text{plim}\ 1/T \sum_{t=1}^{T} R_t(\varphi)$ and, partioning $R_t(\varphi)$ conformably with φ,

$$R_t(\varphi) = -E \begin{pmatrix}
\frac{\partial^2 L_t}{\partial \varphi_1 \partial \varphi_1'} & \frac{\partial^2 L_t}{\partial \varphi_1 \partial \varphi_2'} & \frac{\partial^2 L_t}{\partial \varphi_1 \partial \varphi_3'} \\
\left(\frac{\partial^2 L_t}{\partial \varphi_1 \partial \varphi_2'} \right)' & \frac{\partial^2 L_t}{\partial \varphi_2 \partial \varphi_2'} & \frac{\partial^2 L_t}{\partial \varphi_2 \partial \varphi_3'} \\
\left(\frac{\partial^2 L_t}{\partial \varphi_1 \partial \varphi_3'} \right)' & \left(\frac{\partial^2 L_t}{\partial \varphi_2 \partial \varphi_3'} \right)' & \frac{\partial^2 L_t}{\partial \varphi_3 \partial \varphi_3'}
\end{pmatrix}.$$

The partial derivatives of L_t can be easier computed by rewriting the non-linear model (3.2.2) as:

$$\underbrace{\Phi(L)w_t}_{z_{1t}} = \underbrace{\left[\begin{pmatrix}\triangle x_t' & x_{t-1}'\end{pmatrix} \otimes \Phi(1)\right]}_{Z_{1t}} \underbrace{\begin{pmatrix}\text{vec } \Psi \\ \text{vec } \Pi\end{pmatrix}}_{\varphi_1} + u_t \tag{3.2.15}$$

and equation (3.2.4) as:

$$\underbrace{(w_t - z_t)}_{z_{2t}} = \underbrace{[(w_{t-1} - z_t)' \otimes I_{n-1}]}_{Z_{2t}} \underbrace{\text{vec } \Phi_1}_{\varphi_2} + u_t. \tag{3.2.16}$$

The gradient of L_t respect to $\varphi^{*\prime} = \begin{pmatrix}\varphi_1' & \varphi_2'\end{pmatrix}$ is (Deschamps, 1993):

$$\frac{\partial L_t}{\partial \varphi^*} = \begin{pmatrix} Z_{1t}'\Omega^{-1}u_t \\ Z_{2t}'\Omega^{-1}u_t \end{pmatrix} \tag{3.2.17}$$

and

$$\frac{\partial L_t}{\partial \varphi_3} = -\frac{1}{2}\Omega^{-1} + \frac{1}{2}\Omega^{-1}u_t u_t'\Omega^{-1}. \tag{3.2.18}$$

From equations (3.2.15), (3.2.16) and (3.2.17), we can compute the elements of $R_t(\varphi)$, in particular,

$$\begin{aligned}
\frac{\partial^2 L_t}{\partial \varphi_1 \partial \varphi_1'} &= -Z_{1t}'\Omega^{-1}Z_{1t} \\
\frac{\partial^2 L_t}{\partial \varphi_1 \partial \varphi_2'} &= -Z_{1t}'\Omega^{-1}Z_{2t} \\
\frac{\partial^2 L_t}{\partial \varphi_2 \partial \varphi_2'} &= -Z_{2t}'\Omega^{-1}Z_{2t} \\
\frac{\partial^2 L_t}{\partial \varphi_1 \partial \varphi_3'} &= \frac{\partial^2 L_t}{\partial \varphi_2 \partial \varphi_3'} = 0
\end{aligned}$$

so that if we consider only the first diagonal block of $R(\varphi)$ (the block corresponding to $\varphi^{*\prime} = \begin{pmatrix}\varphi_1' & \varphi_2'\end{pmatrix}$), we have:

$$R_t(\varphi^*) = \begin{pmatrix} Z_{1t}' \\ Z_{2t}' \end{pmatrix} \Omega^{-1} \begin{pmatrix} Z_{1t} & Z_{2t} \end{pmatrix} \tag{3.2.19}$$

and

$$R(\varphi^*) = \text{plim}\ \frac{1}{T}\sum_{t=1}^{T}\begin{pmatrix} Z'_{1t} \\ Z'_{2t} \end{pmatrix}\Omega^{-1}\begin{pmatrix} Z_{1t} & Z_{2t} \end{pmatrix}. \tag{3.2.20}$$

It follows then that:

$$\sqrt{T}\begin{pmatrix} \widehat{\varphi}_1 - \varphi_1 \\ \widehat{\varphi}_2 - \varphi_2 \end{pmatrix} \underset{L}{\Longrightarrow} N\left(0, R^{-1}(\varphi^*)\right). \tag{3.2.21}$$

Resuming, we will find the ML estimators as follows:

(1) Firstly, we compute the initial values for the iteration procedure by

- defining arbitrary values (for example OLS estimators) Ψ^0,Π^0 and Φ_1^0 for Ψ, Π, and Φ_1. Let $\varphi^{*,0} = \begin{pmatrix} \text{vec } \Psi^0 \\ \text{vec } \Pi^0 \\ \text{vec } \Phi_1^0 \end{pmatrix}$.
- computing $\widehat{\Omega}_0$, conditional on Φ_1^0 and Ψ^0 and Π^0, using equations (3.2.11) and (3.2.12):

$$\widehat{\Omega} = \frac{1}{T}\sum_{t=1}^{T}\widehat{u}_t\widehat{u}'_t$$

$$\widehat{u}_t = \widehat{\Phi}(L)w_t - \left[\begin{pmatrix} \triangle x'_t & x'_{t-1} \end{pmatrix} \otimes \widehat{\Phi}(1)\right]\begin{pmatrix} \text{vec } \widehat{\Psi} \\ \text{vec } \widehat{\Pi} \end{pmatrix}$$

- computing the OLS estimator of vec Φ_1^0 using equation (3.2.8):

$$\text{vec } \widehat{\Phi}_1 = \left[\sum_{t=1}^{T}\left[(w_{t-1} - z_t)' \otimes I_{n-1}\right]'\left[(w_{t-1} - z_t)' \otimes I_{n-1}\right]\right]^{-1} \cdot \sum_{t=1}^{T}\left[(w_{t-1} - z_t)' \otimes I_{n-1}\right]'(w_t - z_t)$$

and the GLS estimator of $\begin{pmatrix} \text{vec } \widehat{\Psi} \\ \text{vec } \widehat{\Pi} \end{pmatrix}$ using equation (3.2.10):

$$\begin{pmatrix} \text{vec } \widehat{\Psi} \\ \text{vec } \widehat{\Pi} \end{pmatrix} = \left[\sum_{t=1}^{T} \left[\begin{pmatrix} \triangle x_t' & x_{t-1}' \end{pmatrix} \otimes \widehat{\Phi}(1) \right]' \widehat{\Omega}^{-1} \left[\begin{pmatrix} \triangle x_t' & x_{t-1}' \end{pmatrix} \otimes \widehat{\Phi}(1) \right] \right]^{-1}$$

$$\cdot \sum_{t=1}^{T} \left[\begin{pmatrix} \triangle x_t' & x_{t-1}' \end{pmatrix} \otimes \widehat{\Phi}(1) \right]' \widehat{\Omega}^{-1} \widehat{\Phi}(L) w_t.$$

(2) Secondly, in order to maximize the loglikelihood function, we will compute φ^* by applying the following iteration rule:

$$\varphi^{*,k+1} = \varphi^{*,k} - \lambda_k A_k g(\varphi^{*,k})$$

where $A_k = \left[E\left(\frac{\partial^2 L}{\varphi^* \varphi^{*\prime}} \right) \right]^{-1} = \left[-R(\varphi^*) \right]^{-1}$ ($R(\varphi^*)$ being the information matrix defined by (3.2.20)), $g(\varphi^{*,k}) = \partial L / \partial \varphi^{*,k}$, and λ_k is the scalar which maximizes:

$$F(\lambda_k) = L\left(\varphi^{*,k} - \lambda_k A_k g(\varphi^{*,k}) \right)$$

where L is the loglikelihood defined by (3.2.14).

3.2.2 Restricted ML Estimation

We showed in Section 2.4 that homogeneity implies, in the relative price model, that the last column of Π is a null vector. Thus, maximum likelihood estimators under homogeneity can be obtained by deleting the last column of Π and the last row of x_t. We defined:

$$x_t^* = \begin{pmatrix} 1 & \ln \frac{M_t}{P_t} & \ln \frac{p_{1t}}{p_{nt}} & \ldots & \ln \frac{p_{n-1,t}}{p_{nt}} \end{pmatrix}'$$
$$\triangle x_t^* = \begin{pmatrix} \triangle \ln \frac{M_t}{P_t} & \triangle \ln \frac{p_{1t}}{p_{nt}} & \ldots & \triangle \ln \frac{p_{n-1,t}}{p_{nt}} \end{pmatrix}'$$

$$\Pi^* = \begin{pmatrix} \alpha_1 & \beta_1 & \gamma_{11} & \cdots & \gamma_{1,n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha_{n-1} & \beta_{n-1} & \gamma_{n-1,1} & \cdots & \gamma_{n-1,n-1} \end{pmatrix}$$

$$\Gamma_0^* = \begin{pmatrix} \beta_1^0 & \gamma_{11}^0 & \cdots & \gamma_{1,n-1}^0 \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{n-1}^0 & \gamma_{n-1,1}^0 & \cdots & \gamma_{n-1,n-1}^0 \end{pmatrix},$$

and we wrote the homogeneous model as:

$$\Phi(L)w_t = \Phi(1)\left[\Pi^* x_{t-1}^* + \Psi^* \triangle x_t^*\right] + u_t \tag{3.2.22}$$

$$= \left[\begin{pmatrix} \triangle x_t^* & x_{t-1}^* \end{pmatrix}' \otimes \Phi(1)\right] \begin{pmatrix} \text{vec } \Psi^* \\ \text{vec } \Pi^* \end{pmatrix} + u_t \tag{3.2.23}$$

where $\Psi^* = \Phi^{-1}(1)\Gamma_0^*$. The ML estimators under homogeneity can be computed as the unrestricted estimators, so that:

$$\begin{pmatrix} \text{vec } \widehat{\Psi}^* \\ \text{vec } \widehat{\Pi}^* \end{pmatrix} = \left[\sum_{t=1}^{T}\left[\begin{pmatrix} \triangle x_t^{*\prime} & x_{t-1}^{*\prime} \end{pmatrix} \otimes \widehat{\Phi}(1)\right]' \widehat{\Omega}^{-1}\left[\begin{pmatrix} \triangle x_t^{*\prime} & x_{t-1}^{*\prime} \end{pmatrix} \otimes \widehat{\Phi}(1)\right]\right]^{-1}$$

$$\cdot \sum_{t=1}^{T}\left[\begin{pmatrix} \triangle x_t^{*\prime} & x_{t-1}^{*\prime} \end{pmatrix} \otimes \widehat{\Phi}(1)\right]' \widehat{\Omega}^{-1}\widehat{\Phi}(L)w_t \tag{3.2.24}$$

where:

$$\widehat{\Omega} = \frac{1}{T}\sum_{t=1}^{T}\widehat{u}_t\widehat{u}_t'$$

$$\widehat{u}_t = \widehat{\Phi}(L)w_t - \left[\begin{pmatrix} \triangle x_t^{*\prime} & x_{t-1}^{*\prime} \end{pmatrix} \otimes \widehat{\Phi}(1)\right] \begin{pmatrix} \text{vec } \widehat{\Psi}^* \\ \text{vec } \widehat{\Pi}^* \end{pmatrix}$$

$$\widehat{\Phi}_1 = \left[\sum_{t=1}^{T}(w_t - z_t^*)(w_{t-1} - z_t^*)'\right]\left[\sum_{t=1}^{T}(w_{t-1} - z_t^*)(w_{t-1} - z_t^*)'\right]^{-1} \tag{3.2.25}$$

$$z_t^* = \Pi^* x_{t-1}^* + \Psi^* \triangle x_t^*.$$

Symmetric estimators can be computed by imposing, in the homogeneous model, the following restriction:

$$\begin{pmatrix} \text{vec } \Psi^* \\ \text{vec } \Pi^* \end{pmatrix} = \begin{pmatrix} I_{n(n-1)} & 0 \\ 0 & D_s \end{pmatrix} \begin{pmatrix} \text{vec } \Psi^* \\ \pi_s \end{pmatrix} = D\theta_s \tag{3.2.26}$$

with π_s, θ_s, D_s and D defined by equations (2.4.16), (2.4.17) and (2.4.18). Substituting (3.2.26) in the regression equation (3.2.23), we can compute the ML estimation of θ_s by applying again generalized least squares:

$$\widehat{\theta}_s = \left[\sum_{t=1}^{T} y_t' \widehat{\Omega}^{-1} y_t \right]^{-1} \sum_{t=1}^{T} y_t' \widehat{\Omega}^{-1} \widehat{\Phi}(L) w_t \tag{3.2.27}$$

with:

$$\begin{aligned} y_t &= \left[\left(\triangle x_t^{*\prime} \quad x_{t-1}^{*\prime} \right) \otimes \widehat{\Phi}(1) \right] D \\ \widehat{\Omega} &= \frac{1}{T} \sum_{t=1}^{T} \widehat{u}_t \widehat{u}_t' \\ \widehat{u}_t &= \widehat{\Phi}(L) w_t - \left[\left(\triangle x_t^{*\prime} \quad x_{t-1}^{*\prime} \right) \otimes \widehat{\Phi}(1) \right] D \widehat{\theta}_s \end{aligned}$$

and where $\widehat{\Phi}(1)$ is defined as in the homogeneous model.

From the results at the end of Section 3.2.1, we have:

$$\sqrt{T} \begin{pmatrix} \widehat{\theta}_s - \theta_s \\ \widehat{\varphi}_2 - \varphi_2 \end{pmatrix} \underset{L}{\Longrightarrow} N\left(0, D^* \left[D^{*\prime} R^*(\theta_s, \varphi_2) D^* \right]^{-1} D^{*\prime} \right) \tag{3.2.28}$$

where θ_s are the symmetry constrained estimators of vec Ψ^* and π_s, $\varphi_2 =$ vec Φ_1, $R(\theta_s, \varphi_2)$ is the first diagonal block of the information matrix and

$$D^* = \begin{pmatrix} I_{n(n-1)} & 0 & 0 \\ 0 & D_s & 0 \\ 0 & 0 & I_{(n-1)(n-1)} \end{pmatrix}.$$

We adopted a general presentation here focused on the error-correction model and not on each of the models we will employ in the empirical part of our work. It is obvious that most of the formulas discussed in this section simplify when applied to the static, autoregressive-errors or partial-adjustment models. In the following section, we will briefly present the particularities of each of these three models and their consequences for estimation.

3.2.3 Unrestricted and Restricted Estimation of the Static, Autoregressive-errors and Partial-adjustment Models

Static Model: It is the simplest model to estimate since it does not include the autoregressive coefficient Φ_1. We saw that it can be derived from the error-correction model by imposing $\Phi(L) = I$ and $\Gamma(L) = \Gamma_0$. In vec form, it can be written as:

$$w_t = (x_t' \otimes I_{n-1}) \text{ vec } \Pi + u_t \tag{3.2.29}$$

and the unconstrained model can be estimated by direct ML as:

$$\text{vec } \widehat{\Pi} = \left[\sum_{t=1}^{T} (x_t' \otimes I_{n-1})'(x_t' \otimes I_{n-1}) \right]^{-1} \sum_{t=1}^{T} (x_t' \otimes I_{n-1})' w_t.$$

Homogeneous estimation requires that the last column of Π is a null vector while symmetric estimation can be obtained by imposing vec $\Pi = D_s \pi_s$ where $\pi_s = \begin{pmatrix} \alpha_1 & \dots & \alpha_{n-1} & \beta_1 & \dots & \beta_{n-1} & \gamma_{11} & \gamma_{21} & \dots & \gamma_{n-1,n-1} \end{pmatrix}'$ is the vector of unconstrained coefficients and:

$$D_s = \begin{pmatrix} I_{2(n-1)} & 0 \\ 0 & L \end{pmatrix}$$

is the duplication matrix. The restricted model, under both homogeneity and symmetry, has been written as:

$$w_t = (x_t^{*\prime} \otimes I_{n-1}) D_s \pi_s + u_t \tag{3.2.30}$$

and the ML estimator of π_s will be:

$$\begin{aligned}\widehat{\pi}_s &= \left[\sum_{t=1}^{T} D_s'(x_t^{*\prime} \otimes I_{n-1})'(x_t^{*\prime} \otimes I_{n-1})D_s\right]^{-1} \sum_{t=1}^{T} D_s'(x_t^{*\prime} \otimes I_{n-1})'w_t \\ &= \left[D_s' \sum_{t=1}^{T}(x_t^{*} x_t^{*\prime} \otimes I_{n-1})D_s\right]^{-1} D_s' \sum_{t=1}^{T}(x_t^{*\prime} \otimes I_{n-1})'w_t. \end{aligned} \tag{3.2.31}$$

In order to impose homogeneity only, it is sufficient to set $D_s = I_{(n-1)(n+1)}$.

Autoregressive-errors Model: In Section 2.4.3, we showed that the autoregressive-errors model:

$$w_t = \Pi x_t + v_t \quad \text{with} \tag{3.2.32}$$

$$v_t = \Phi_1 v_{t-1} + u_t, \tag{3.2.33}$$

could be rewritten as:

$$\Phi(L)w_t = \Pi x_t - \Phi_1 \Pi x_{t-1} + u_t \tag{3.2.34}$$

or in vec form:

$$\Phi(L)w_t = \left[(x_t' \otimes I_{n-1}) - (x_{t-1}' \otimes \Phi_1)\right] \text{ vec } \Pi + u_t, \tag{3.2.35}$$

so that, given Φ_1, it is a multivariate regression with coefficient vector Π. But equation (3.2.34) can also be rewritten as:

$$w_t - \Pi x_t = \Phi_1(w_{t-1} - \Pi x_{t-1}) + u_t \tag{3.2.36}$$

which, given Π, is a linear model with coefficient matrix Φ_1. We can then use the procedure presented for the general error-correction model and estimate Φ_1 (conditional on Π) as:

$$\widehat{\Phi}_1 = \sum_{t=1}^{T}(w_t - \Pi x_t)(w_{t-1} - \Pi x_{t-1})' \left[\sum_{t=1}^{T}(w_{t-1} - \Pi x_{t-1})(w_{t-1} - \Pi x_{t-1})'\right]^{-1}. \tag{3.2.37}$$

Upon substituting $\widehat{\Phi}_1$ into equation (3.2.35), we can compute the ML estimator of vec Π by applying generalized least squares:

$$\operatorname{vec}\widehat{\Pi} = \left[\sum_{t=1}^{T}\left[(x_t' \otimes I_{n-1}) - (x_{t-1}' \otimes \widehat{\Phi}_1)\right]' \widehat{\Omega}^{-1}\left[(x_t' \otimes I_{n-1}) - (x_{t-1}' \otimes \widehat{\Phi}_1)\right]\right]^{-1} \cdot \sum_{t=1}^{T}\left[(x_t' \otimes I_{n-1}) - (x_{t-1}' \otimes \widehat{\Phi}_1)\right]' \widehat{\Omega}^{-1}\widehat{\Phi}(L)w_t \quad (3.2.38)$$

where:

$$\widehat{\Omega} = \frac{1}{T}\sum_{t=1}^{T}\widehat{u}_t\widehat{u}_t'$$

$$\widehat{u}_t = \widehat{\Phi}(L)w_t - \left[(x_t' \otimes I_{n-1}) - (x_{t-1}' \otimes \widehat{\Phi}_1)\right] \operatorname{vec}\widehat{\Pi}.$$

As for the static model, constrained estimation requires, under homogeneity, that the last column of Π is a null vector and, under symmetry, that vec $\Pi = D_s\pi_s$ where, again, π_s is the vector of unconstrained coefficients and D_s is the duplication matrix. The constrained model can then be written as:

$$\Phi(L)w_t = \left[(x_t^{*\prime} \otimes I_{n-1}) - (x_{t-1}^{*\prime} \otimes \Phi_1)\right] D_s\pi_s + u_t \quad (3.2.39)$$

and the ML estimator of π_s is:

$$\widehat{\pi}_s = \left[\sum_{t=1}^{T} y_t'\widehat{\Omega}^{-1}y_t\right]^{-1} \sum_{t=1}^{T} y_t'\widehat{\Omega}^{-1}\widehat{\Phi}(L)w_t \quad (3.2.40)$$

where $y_t = \left[(x_t^{*\prime} \otimes I_{n-1}) - (x_{t-1}^{*\prime} \otimes \widehat{\Phi}_1)\right] D_s$ and $\widehat{\Omega}$, $\widehat{u}_t$ are defined consequently. In order to apply homogeneity only, set $D_s = I_{(n-1)(n+1)}$.

Partial-adjustment Model: With $p = q = 1$, the partial-adjustment model has been written as:

$$\Phi(L)w_t = \Phi(1)\Pi x_t + u_t \quad (3.2.41)$$

or in vec form:

$$\Phi(L)w_t = [x_t' \otimes \Phi(1)] \text{ vec } \Pi + u_t. \tag{3.2.42}$$

The model can be equivalently rewritten as:

$$\begin{aligned} w_t - \Phi_1 w_{t-1} &= \Pi x_t - \Phi_1 \Pi x_t + u_t \\ w_t - \Pi x_t &= \Phi_1 [w_{t-1} - \Pi x_t] + u_t \end{aligned} \tag{3.2.43}$$

so that, again, we have a bi-linear model where, conditional on Φ_1, equation (3.2.42) is a multivariate model with coefficient vector vec Π and equation (3.2.43) is, conditional on Π, a reduced form with coefficient matrix Φ_1. Following the procedure presented in Section 3.2.1, we can compute the ML estimator of Φ_1 as:

$$\widehat{\Phi}_1 = \sum_{t=1}^{T} (w_t - \Pi x_t)(w_{t-1} - \Pi x_t)' \left[\sum_{t=1}^{T} (w_{t-1} - \Pi x_t)(w_{t-1} - \Pi x_t)' \right]^{-1} \tag{3.2.44}$$

and the ML estimator of Π as:

$$\begin{aligned} \text{vec } \widehat{\Pi} = & \left[\sum_{t=1}^{T} \left[x_t' \otimes \widehat{\Phi}(1) \right]' \widehat{\Omega}^{-1} \left[x_t' \otimes \widehat{\Phi}(1) \right] \right]^{-1} \\ & \cdot \sum_{t=1}^{T} \left[x_t' \otimes \widehat{\Phi}(1) \right]' \widehat{\Omega}^{-1} \widehat{\Phi}(L) w_t \end{aligned} \tag{3.2.45}$$

where:

$$\begin{aligned} \widehat{\Omega} &= \frac{1}{T} \sum_{t=1}^{T} \widehat{u}_t \widehat{u}_t' \\ \widehat{u}_t &= \widehat{\Phi}(L) w_t - \left(x_t' \otimes \widehat{\Phi}(1) \right) \text{ vec } \widehat{\Pi}. \end{aligned}$$

Homogeneous estimations can be obtained by deleting the last column of Π and the last row of x_t while symmetric estimations can be computed by imposing vec $\Pi = D_s \pi_s$ where D_s is defined as above and π_s is the vector of unconstrained coefficients of Π. The restricted model can be written as:

$$\Phi(L) w_t = [x_t^{*\prime} \otimes \Phi(1)] D_s \pi_s + u_t \tag{3.2.46}$$

and the homogeneous and symmetric estimator is:

$$\widehat{\pi}_s = \left[\sum_{t=1}^{T} D_s' \left[x_t^{*\prime} \otimes \widehat{\Phi}(1) \right]' \widehat{\Omega}^{-1} \left[x_t^{*\prime} \otimes \widehat{\Phi}(1) \right] D_s \right]^{-1} \sum_{t=1}^{T} D_s' \left[x_t^{*\prime} \otimes \widehat{\Phi}(1) \right]' \widehat{\Omega}^{-1} \widehat{\Phi}(L) w_t. \tag{3.2.47}$$

The homogeneous estimators can be obtained by setting $D_s = I_{(n-1)(n+1)}$.

We presented a general estimation procedure here for the error-correction model and its application to the static, autoregressive-errors and partial-adjustment models. Once estimated the model, the further step of our analysis will be to test the imposed restrictions. This will be done using the likelihood ratio statistics presented in the following section.

3.3 Hypothesis Testing

Two groups of hypotheses will be tested: the first one concerns the regularity restrictions (homogeneity and symmetry) while the second one focusses on the dynamic specification of the model. Our first objective is to check if the regularity assumptions are rejected by data, which has often been the case in previous empirical analysis. We will compute these tests for each of the models presented. The second objective is to test whether or not the restrictions we imposed on the dynamic specification are significant, i.e. if one, or more, of the "simplified models" (the static, autoregressive-errors and partial-adjustment models) may represent the dynamics of our general dynamic import allocation model.

In order to compute the tests we need to choose an appropriate test statistic. Between the three well known principles for the hypothesis test construction, i.e. the Wald test (W), the Lagrange multiplier (LM) method and the maximum likelihood ratio (LR), we decided to use the latter one. This choice seems to be the natural one since we estimate both the unrestricted and restricted models by maximum likelihood techniques.

The LR test is based upon the difference between the maximized loglikelihood under the null hypothesis (the constrained model) and under the alternative hypothesis (the unconstrained model). Let φ and φ^0 respectively be the vector of parameters that maximize the unconstrained and constrained likelihood. The loglikelihood ratio statistic is defined as:

$$LR = -2\left(L(\varphi^0) - L(\varphi)\right) \tag{3.3.1}$$

and follows asymptotically, under the null hypothesis, a χ^2_r distribution, where r is the number of restrictions (for a proof see for example Amemiya, 1985) .

Using this technique we can test all the hypotheses in which we are interested:

1. homogeneity and symmetry in the error-correction, static, autoregressive-errors and partial-adjustment models;

2. the restrictions imposed on the error-correction model to obtain the restricted dynamic specifications, namely:

   (a) $\Phi(L) = I$ (or $\Phi_j = 0$ for $j > 0$), and $\Gamma(L) = \Gamma_0$ (or $\Gamma_k = 0$ for $k > 0$) for the static model;

(b) $\Gamma(L) = \Phi(L)\Gamma_0$ for the autoregressive-errors model;

(c) $\Gamma_i = 0$ for $i > 0$ for the partial-adjustment model.

Chapter 4

Data Analysis

4.1 Data Description

The data set contains monthly observations from January 1988 to December 2002 on import budget shares of manufactures, import prices and real expenditure for Switzerland and from four different sources (Germany, France, Italy and USA). The data were collected from the Swiss-impex database of the Swiss Federal Custom Administration.

Manufactures are defined as SITC (Standard International Trade Classification) sections 5-8, which includes chemical and related products, manufactured goods, machinery and transport equipment and other miscellaneous manufactured articles (see Appendix A for details). The chosen countries are the major sources of imports over the considered period, they account for around 70% of total Swiss manufactured imports. Germany ensures more than a third of them, France around $12-15\%$, Italy $10-12\%$ and the USA between 8 and 10%. The number of countries has been restricted to four in order to avoid the problem of lack of degrees of freedom.[1]

[1]In what follows, we refer only to imports from these countries, defining total imports as the sum of imports from Germany, France, Italy and the USA, and budget shares as the ratio of imports from every source over that total.

Year	**Germany** (shares)	**France** (shares)	**Italy** (shares)	**USA** (shares)	**Total of imports** (CHF in millions)
1988	53.42	19.45	16.89	10.24	42'891
1989	54.97	19.08	16.96	8.99	44'148
1990	55.60	18.23	17.18	8.98	44'475
1991	56.43	17.59	16.81	9.17	49'719
1992	55.02	17.82	16.67	10.49	57'979
1993	54.84	18.05	17.13	9.98	59'316
1994	53.73	17.85	16.38	12.03	57'941
1995	55.16	17.84	16.52	10.48	55'981
1996	54.74	18.37	16.44	10.45	53'427
1997	54.84	18.35	16.52	10.29	55'445
1998	54.89	18.54	16.50	10.07	58'034
1999	51.67	19.01	17.66	11.66	58'736
2000	51.33	18.65	16.67	13.35	64'847
2001	52.67	18.77	16.68	11.88	67'052
2002	52.47	20.31	16.37	10.85	70'997

Table 4.1: Swiss purchases of manufactures (annual budget shares and total imports, 1988-2002)

Table 4.1 summarizes annual budget shares of the four countries (the last column is the sum of the imports from those countries). Considering only the four countries, Germany provides more than half of total imports, France around one fifth, Italy slightly less and the USA around 10%. It is interesting to notice that, while imports in value grow fast over the period (total imports increase by more that 65%), countries shares tend to be stationary. This can be clearly observed in Figure 4.1.

Import prices, for each country, were computed as Stone's price indexes using data at the lower, and more detailed, level (5 digits) of the SITC classification, defining expenditures shares on "items" as weights and the logarithm of their unit values as prices. For example, the logarithm of the import price for Germany at time t, is equal to the weighted average of the unit values logarithms, at time t, of all SITC (sections 5-8) sub-categories.

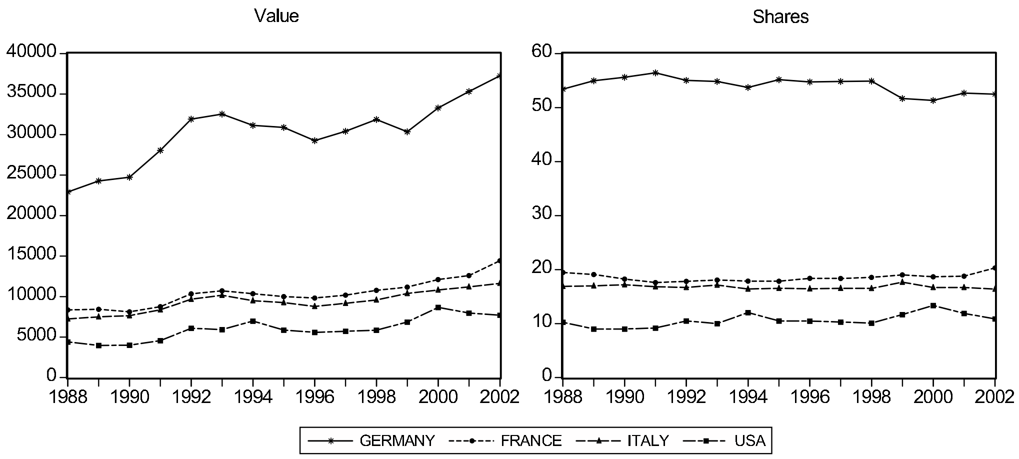

Figure 4.1: Annual imports of manufactured goods from Germany, France Italy and the USA (CHF in millions and budget shares, 1988-2002)

The logarithm of real expenditure was computed as the difference between the logarithm of total expenditure on imports (the sum of imports from the four countries) at time t, M_t, with a Stone price index defined as $\log P_t = \sum_{k=1}^{4} w_{kt} \log p_{kt}$ where w_{kt} denotes the budget share and p_{kt} the import price of country k at time t. Annual price averages and real expenditure are plotted in Figure 4.2.

Over the whole period, the import prices follow an upward trend, but with some fluctuations. US prices seem to rise faster than the others, especially in the last 10 years. A similar growth characterizes French and Italian data, while German prices remained almost steady. A much more irregular pattern is followed by the real expenditure term, with a sharp rise at the beginning of the period, followed by a fall and a recovery at the end of the '90s before another fall in the last observed year.

Figures 4.1 and 4.2 give us an idea of the trend followed by the series, abstracting from all monthly fluctuations. We noticed an upward trend, especially in import prices, but this is, of course, not enough to carry on an analysis of data stationarity. We will explain in the following sections the relevance of stationarity for the

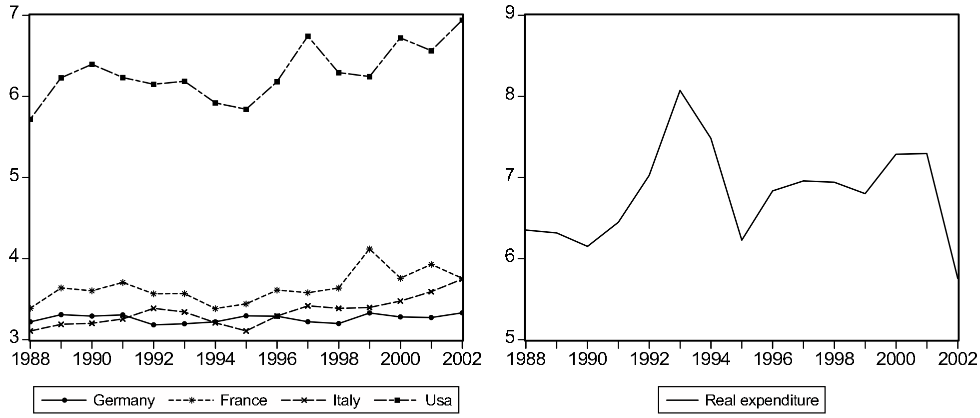

Figure 4.2: Import prices and real expenditure (annual averages, 1988-2002)

equilibrium multipliers estimation. We will also consider several data specifications (levels, first-differences and seasonal differences) and introduce the test statistics we will employ in order to test stationarity.

An important factor that has not been mentioned and could contribute in explaining the evolution of prices (and shares) are exchange rates. Budget shares and prices are evaluated in Swiss francs, this implies that any variation in exchange rates reflects in a variation in prices and shares.

4.2 Stationarity and the Equilibrium Multipliers Identification

The main goal of our analysis is to estimate the equilibrium multipliers Π, i.e. the coefficients of the long-run structure of our models. It is clear that such a relationship exists only if our VAR process converges to an equilibrium level. In this section we will discuss the conditions under which the equilibrium multipliers exist and are identifiable.

Let's rewrite the general dynamic model:

$$\Phi(L)w_t = \Gamma(L)x_t + u_t, \tag{4.2.1}$$

with $w_t = \begin{pmatrix} w_{1t} & \dots & w_{n-1,t} \end{pmatrix}'$ and $x_t = \begin{pmatrix} 1 & \ln\, p_{1t} & \dots & \ln\, p_{nt} & \ln\, M_t/P_t \end{pmatrix}'$ by applying the lemma presented in Section 2.3.1 to both $\Phi(L)$ and $\Gamma(L)$:

$$\Phi(1)w_{t-1} + \Phi^*(L)\triangle w_t = \Gamma(1)x_{t-1} + \Gamma^*(L)\triangle x_t + u_t$$

with $\triangle x_t = \begin{pmatrix} \triangle \ln\, p_{1t} & \dots & \triangle \ln\, p_{nt} & \triangle \ln\, M_t/P_t \end{pmatrix}'$. Assuming that the VAR is covariance-stationary, we take expectations of both sides:

$$\Phi(1)E(w_{t-1}) + \Phi^*(L)E(\triangle w_t) = \Gamma(1)E(x_{t-1}) + \Gamma^*(L)E(\triangle x_t) + E(u_t)$$

but, at the equilibrium levels, $E(\triangle w_t) = E(\triangle x_t) = E(u_t) = 0$ so that:

$$E(w_t) = \Phi^{-1}(1)\Gamma(1)E(x_t) = \Pi E(x_t).$$

The matrix Π is then the matrix that links the equilibrium levels of w and x, if, of course, they exist.

The crucial assumption that allows us to compute the equilibrium multipliers is, then, that the vector process generating w_t is covariance-stationary, which in turn requires that $\Phi(L)$ does not contain unit roots (and that the exogenous process generating x_t is covariance-stationary). In fact, the presence of unit roots in $\Phi(L)$ would imply that $\Phi(1)$ is singular (Hamilton, 1994) and that the regressor matrix of the VECM associated to (4.2.1) (cf. Sections 2.3.1 and 3.2):

$$\Phi^*(L)\triangle w_t = -\Phi(1)[w_{t-1} - \Pi x_{t-1}] + \Gamma^*(L)\triangle x_t + u_t \tag{4.2.2}$$

or, in vec form,

$$\Phi^*(L)w_t = \left[\begin{pmatrix} \triangle x_t' & x_{t-1}' \end{pmatrix} \otimes \Phi(1)\right] \begin{pmatrix} \text{vec } \Psi \\ \text{vec } \Pi \end{pmatrix} + u_t \tag{4.2.3}$$

with $\Psi = \Phi^{-1}(1)\Gamma^*(L)$ and $\Pi = \Phi^{-1}(1)\Gamma(1)$, does not have full column rank, so that the elements of Π (and Ψ) are not identifiable (Deschamps, 1998).

But what are the conditions ensuring that a VAR process is stationary? Hamilton (1994) defines a covariance-stationary process as an arbitrary process z_t of the form:

$$z_t = c + \theta_1 z_{t-1} + \theta_2 z_{t-2} + \ldots + \theta_p z_{t-p} + u_t \quad \text{or} \quad \Theta(L) z_t = c + u_t,$$

with $\Theta(L) = I - \theta_1 L - \theta_2 L^2 - \ldots - \theta_p L^p$, whose first and second moments ($E(z_t)$ and $E(z_t z'_{t-1})$) are independent of the date t.

Our general dynamic model:

$$w_t = \Phi_1 w_{t-1} + \Phi_2 w_{t-2} + \ldots + \Phi_p w_{t-p} + \Gamma(L) x_t + u_t$$

is composed of an autoregressive and an exogenous process (each one with its own lag distribution) and Hamilton's proposition can not be directly applied. Therefore, we have to rewrite the model in a way in which the conditions for stationarity appear in a simpler way. Assume first that the exogenous vector x_t is generated by an arbitrary covariance-stationarity VAR of the form:

$$x_t = c + \theta_1 x_{t-1} + \theta_2 x_{t-2} + \ldots + \theta_q x_{t-q} + v_t$$

and let:

$$\mu_t = E(w_t \mid x_t x_{t-1} \ldots) = \Phi^{-1}(L)\Gamma(L) x_t$$

be the expectation of w_t conditional on x_t and its past values. We can rewrite the general dynamic model (4.2.1) in terms of deviations from the conditional expectation as:

$$\begin{aligned}
\Phi(L)w_t &= \underbrace{\Gamma(L)x_t}_{=\Phi(L)\mu_t} + u_t \\
(I - \Phi_1 L - \Phi_2 L^2 - \ldots - \Phi_p L^p)w_t &= (I - \Phi_1 L - \Phi_2 L^2 - \ldots - \Phi_p L^p)\mu_t + u_t \\
w_t - \mu_t &= \Phi_1(w_{t-1} - \mu_{t-1}) + \ldots + \Phi_p(w_{t-p} - \mu_{t-p}) + u_t \\
&= \sum_{j=1}^{p} \Phi_j(w_{t-j} - \mu_{t-j}) + u_t. \qquad (4.2.4)
\end{aligned}$$

Moreover, by defining:

$$\xi_t = \begin{pmatrix} w_{t-p} - \mu_{t-p} \\ \\ w_{t-p+1} - \mu_{t-p+1} \\ \vdots \\ w_t - \mu_t \end{pmatrix}, \quad F = \begin{pmatrix} 0 & I_{n-1} & 0 & \ldots & 0 \\ 0 & 0 & I_{n-1} & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & I_{n-1} \\ \Phi_p & \Phi_{p-1} & \Phi_{p-2} & \ldots & \Phi_1 \end{pmatrix}$$

and

$$\varepsilon_t = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ u_t \end{pmatrix}$$

we can express (4.2.4), and consequently the VAR(p) general dynamic model (4.2.1), as a VAR(1) process:

$$\xi_t = F\xi_{t-1} + \varepsilon_t \qquad (4.2.5)$$

where:

$$E(\varepsilon_t \varepsilon_s') = \begin{cases} \Sigma & \text{for } t = s \\ 0 & \text{otherwise} \end{cases}$$

and:

$$\Sigma = \begin{pmatrix} 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & \Omega \end{pmatrix} \quad \text{with } \Omega = E(u_t u_t').$$

Hamilton (1994) showed that a VAR(1) process of the form of (4.2.5) is covariance-stationary if the eigenvalues of F all lie inside the unit circle. Hence, a VAR(p) is covariance-stationary as long as $\mid \lambda \mid < 1$ for all values of λ that satisfy:

$$\mid I_{n-1}\lambda^p - \Phi_1\lambda^{p-1} - \Phi_2\lambda^{p-2} - \dots - \Phi_p \mid = 0$$

or, equivalently, if all values of z satisfying:

$$\mid I_{n-1} - \Phi_1 z - \Phi_2 z^2 - \dots - \Phi_p z^p \mid = 0$$

lie outside the unit circle.

Summarizing, in order to ensure the identification of the equilibrium multipliers matrix Π, our series (endogenous and exogenous) need to be covariance-stationary. This is the reason why, in the next section, we will start our analysis by presenting the data in different forms (seasonal adjusted and unadjusted levels, first-differences and seasonal differences). Afterwards, we will test whether the series are stationary or not. Those that contain unit roots will not be considered in the sequel of the work.

4.3 Data Specification

We highlighted in the previous section the relevance of stationarity for the equilibrium multipliers estimation. If the series are non-stationary, a natural transformation will be to take the first-differences, i.e. to compute:

$$\triangle z_t = z_t - z_{t-1}$$

where z_t is an arbitrary vector. If the series in levels contain unit roots, first-differences will be stationary.

However, stationarity is not the only problem we can encounter working with data. Many economic series tend to follow a seasonal pattern over the course of the year, this is particularly true for monthly data. In Figures 4.3 and 4.4 budget shares, import prices and real expenditure by month are plotted (in levels and first-differences) with their respective monthly means.

A seasonal pattern clearly appears for the four budget shares series, especially during summer months. Seasonality seems to be smaller for the exogenous series, where only French and US prices display significant variations in the means over the year.

This pattern may be the result of social customs (like summer holidays), the interaction with other economic sectors, a particular economic situation (imports are particularly sensible to variations in the economic activity), etc., and can be one of the major factors of variation and instability in data. Seasonality in economic series is viewed in two different ways. One view is that it is a part of many economic series and, when present, one should attempt to explain it. The other view is that seasonality is simply a type of noise that contaminates data and, thus, one should use seasonally adjusted data (Davidson and MacKinnon, 1993).

Seasonality can then be a source of non-stationarity, this is the reason why we will also compute seasonally adjusted data and check whether the stationarity test

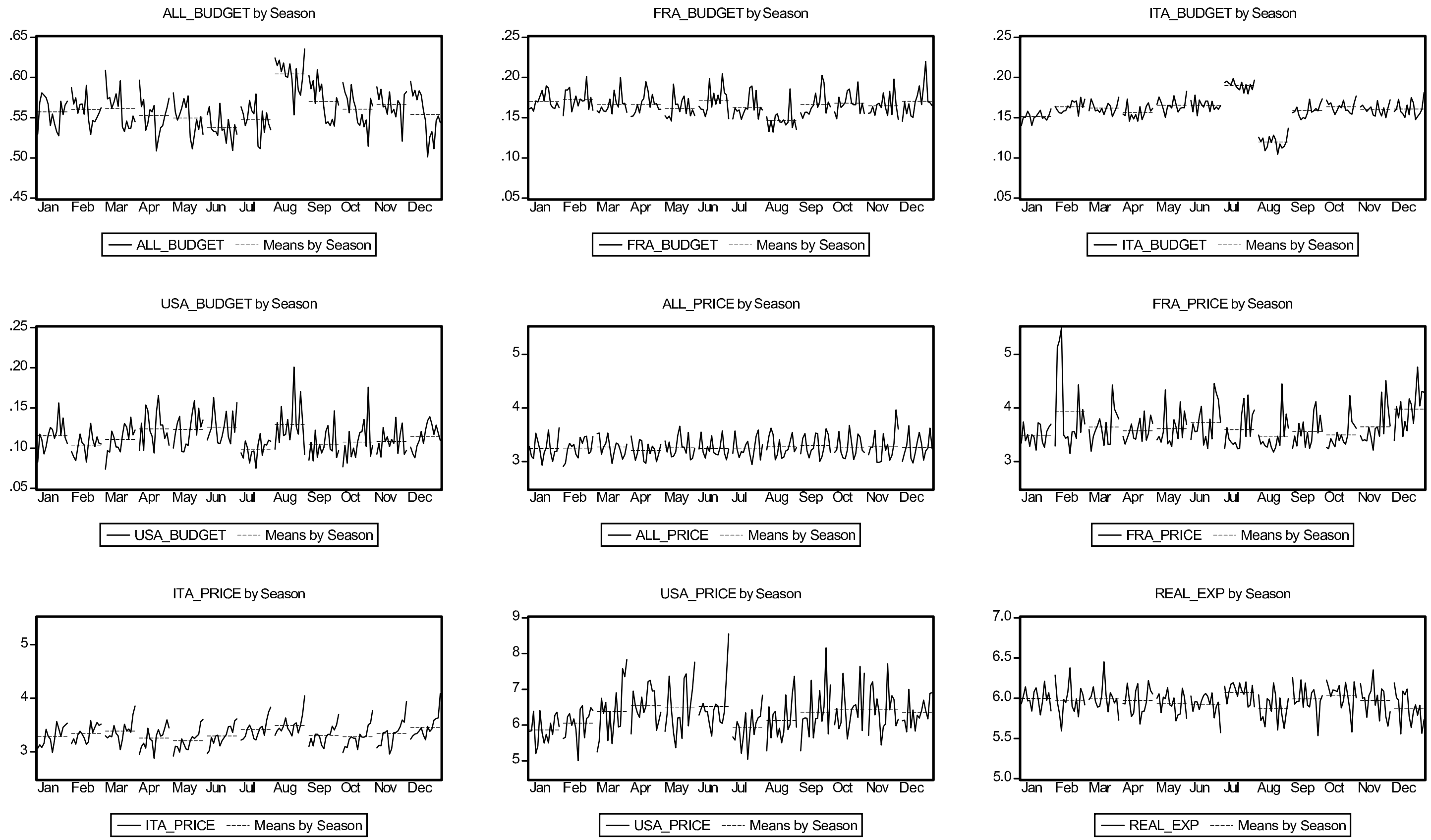

Figure 4.3: Budget shares, import prices and real expenditure by month (levels, 1988-2002)

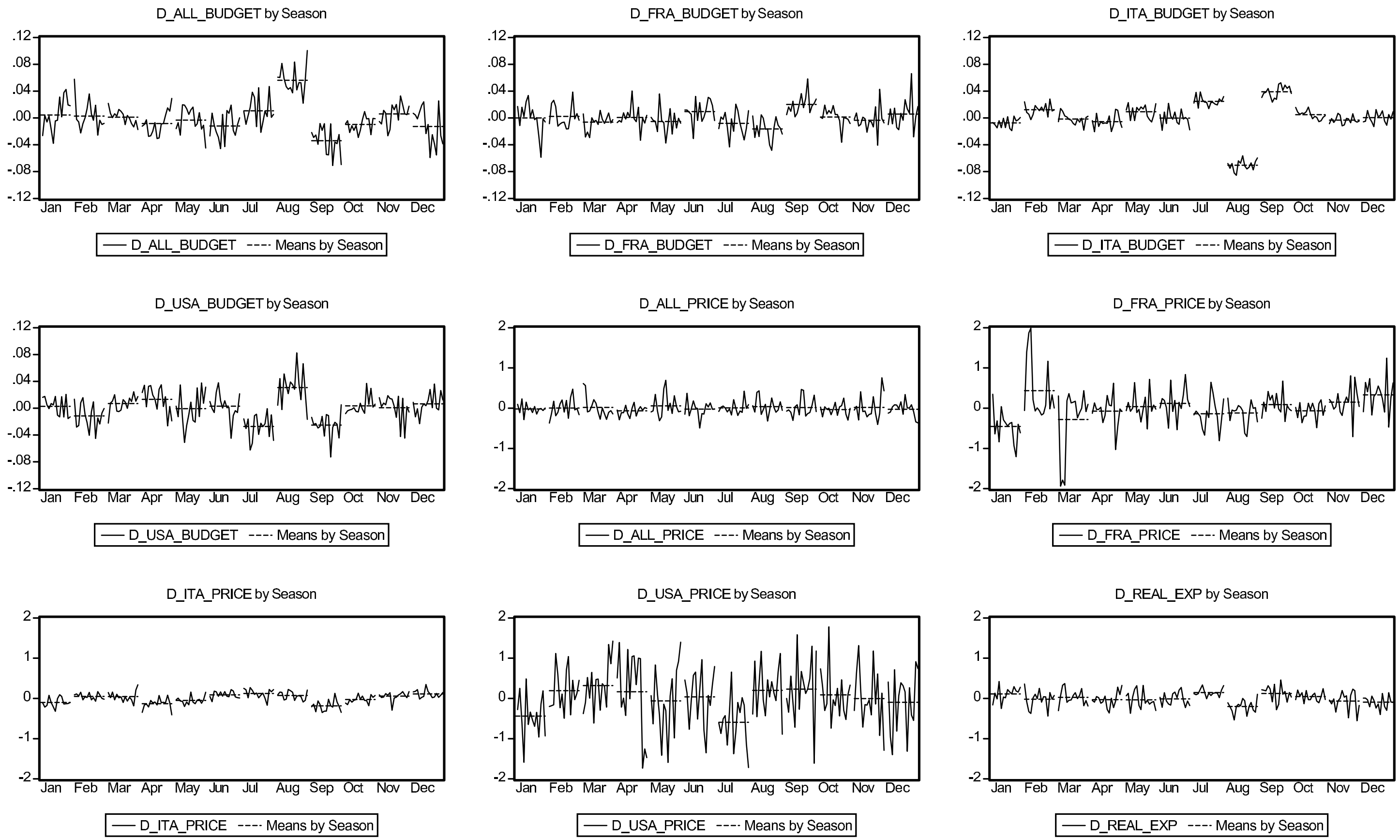

Figure 4.4: Budget shares, import prices and real expenditure by month (first-differences, 1988-2002)

results differ from the ones with unadjusted data. Adjusted data (in levels and first-differences) will be computed using the US Census Bureau's X12 program.[2] This procedure is more complicated, but more powerful (it deals with trend cycles, time-varying seasonality, etc.) than, for example, the use of dummy variables. Moreover, preliminary results with simpler adjustment procedures highlighted serious problems of heteroskedasticity.

A third possible data transformation, that in some way combines the first two, is to compute seasonal differences defined as:

$$\triangle_s z_t = z_t - z_{t-12}.$$

Instead of taking first-differences, i.e. the difference between an observation at time t and the observation on the previous month, we take differences with the observation on the same month of the previous year. This transformation is justified by the hypothesis that the series contain seasonal instead of unit roots. If, as it appears clearly in Figures 4.3 and 4.4, the data are non-stationary and follow a seasonal pattern, computing seasonal differences has the effect of, on one hand, adjusting for seasonality and, on the other hand, eliminating seasonal roots. The advantage of this procedure is that two major sources of non-stationarity are removed in one step. The disadvantage is that at the same time we loose long-term and "seasonal" information.

Seasonal differences are plotted in Figure 4.5. It must be noted that monthly means are all around zero and thus that the seasonal patterns have disappeared. Nevertheless, we will also compute seasonally adjusted data, using the Census X12 procedure, and check whether there are significant differences in the results.

[2] The X12 seasonal adjustment program is publicly provided by the Census. We used the front-end for accessing the X12 program from within EViews 4.1 to compute seasonally adjusted data.

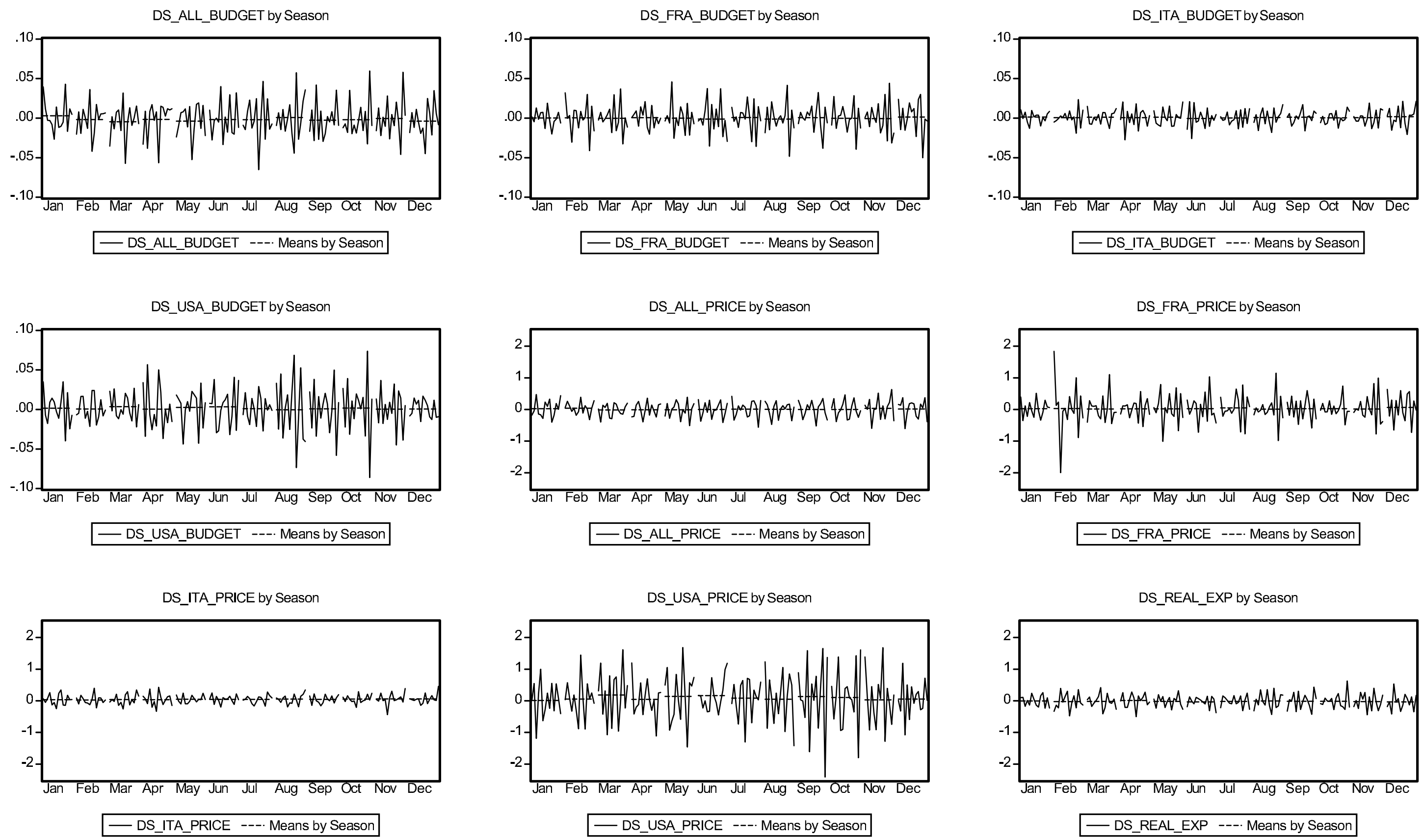

Figure 4.5: Budget shares, import prices and real expenditure by month (seasonal differences, 1988-2002)

In conclusion, we presented six different data specifications in this section (summarized in Table 4.2). For each of the data definitions, we will test the presence of unit roots using Johansen's test. The approach and the test statistics will be presented in the following section.

	Definition	Seasonally unadjusted	Seasonally adjusted
Levels	z_t	⟨variable_name⟩	⟨variable_name⟩_sa
First-differences	$\triangle z_t = z_t - z_{t-1}$	d_⟨variable_name⟩	d_⟨variable_name⟩_sa
Seasonal-differences	$\triangle_s z_t = z_t - z_{t-12}$	ds_⟨variable_name⟩	ds_⟨variable_name⟩_sa

Table 4.2: Data definitions and their denominations

4.4 Joint Stationarity and Johansen's Cointegration Test

We discussed stationarity in detail, however, what we require is not stationarity of the individual variables but stationarity of vectors of variables. The relevant concept is the one of joint stationarity, i.e. if there are unit roots in the joint process generating the vector of variables under analysis. Following Deschamps (1998), joint stationarity will be tested with the help of Johansen's cointegration test.

Assume that an $(n \times 1)$ vector of variables z_t is generated by a VAR process of the form $\Theta(L)z_t = \mu + \delta t + u_t$ with $\Theta(L) = I - \theta_1 L - \ldots - \theta_p L^p$. The VAR can be reparametrized (using the lemma presented in Section 2.3.1) as:

$$\triangle z_t = \mu + \delta t + \Theta(1)z_{t-1} - \sum_{j=1}^{p-1} \theta_j^* \triangle z_{t-j} + u_t \tag{4.4.1}$$

where $\theta_j^* = -\sum_{s=j+1}^{p} \theta_s$ and $\theta_0^* = \theta_0 = I$ and with $E(u_t) = 0$ and $E(u_t u_s') = \Omega$ for $t = s$, 0 otherwise. If we assume that the first differences $\triangle z_t$ are stationary, then every term in equation (4.4.1) is stationary and so must be $\Theta(1)z_{t-1}$. This is the case when $\Theta(1)$ has full rank, and consequently z_{t-1} is stationary. In fact, the matrix $\Theta(1)$ determines whether or not, and to what extent, the system is cointegrated.

Remember that a $(n \times 1)$ vector z_t is said to be cointegrated if each of its elements is integrated of order one and if there exists a nonzero $(n \times 1)$ vector a such that $a'z_t$ is stationary (Hamilton, 1994). There may be up to $h < n$ linearly independent vectors $(a_1, a_2, \ldots, a_h)$ such that $A'z_t$ is a stationary $(h \times 1)$ vector, where:

$$A' = \begin{pmatrix} a'_1 \\ \vdots \\ a'_h \end{pmatrix}.$$

The vectors $(a_1, a_2, \ldots, a_h)$ are not unique; if a_i is a cointegrating vector such that $a'_i z_t$ is stationary, so is ba_i, for any nonzero scalar b, since $ba'_i z_t$ is also stationary. This implies that the $n \times 1$ vector $\vartheta = b'A'$, where b is a nonzero $h \times 1$ vector, could also be described as a cointegrating vector. Letting ϑ' be any row of $\Theta(1)$, we can apply this reasoning to each of the rows of $\Theta(1)$ and it follows that an $n \times h$ matrix B such that $\Theta(1) = BA'$ exists.

Assume that the disturbances u_t in the VAR (4.4.1) are normal and that we have a sample of $T + p$ observations on z (denoted $z_{-p}, \ldots, z_T$). Then, the loglikelihood of $(z_1, z_2, \ldots, z_T)$ conditional on $(z_{-p+1}, z_{-p+2}, \ldots, z_0)$ is given by:

$$\begin{aligned} L(\Omega, \theta_1, \ldots, \theta_{p-1}, \delta, \mu, \Theta(1)) = \frac{-Tn}{2}\log(2\Pi) - \frac{T}{2}\log \mid \Omega \mid -\frac{1}{2}\sum_{t=1}^{T}\Bigg[(\triangle z_t - \mu \\ - \delta t - \Theta(1)z_{t-1} + \sum_{j=1}^{p-1}\theta_j^* \triangle z_{t-j})'\Omega^{-1}(\triangle z_t - \mu - \delta t - \Theta(1)z_{t-1} + \sum_{j=1}^{p-1}\theta_j^* \triangle z_{t-j})\Bigg] \end{aligned} \tag{4.4.2}$$

and the objective of Johansen's approach is to choose $(\Omega, \theta_1, \ldots, \theta_{p-1}, \delta, \mu, \Theta(1))$ in order to maximize (4.4.2) subject to the constraint that there exists h cointegration relationships, i.e. that $\Theta(1) = BA'$.

His approach consists of two steps: the estimation of a set of auxiliary regressions and, from these, the computation of canonical correlations. Let's first regress each element $\triangle z_{it}$ of $\triangle z_t$ on a constant, a time trend and all the elements of the vectors $\triangle z_{t-1}, \ldots, \triangle z_{t-p-1}$ by ordinary least squares and collect them in vector form as:

$$\triangle z_t = \hat{\upsilon}_0 + \hat{\upsilon}_1 t + \sum_{i=1}^{p-1} \hat{\Upsilon}_i \triangle z_{t-i} + \hat{\eta}_t \tag{4.4.3}$$

where $\hat{\Upsilon}$ is the $n \times n$ matrix of OLS estimates and $\hat{\eta}_t$ the $n \times 1$ vector of residuals. Let's also estimate a second set of regressions, where $z_{i,t-1}$ is the independent variable and $\triangle z_{t-1}, \ldots, \triangle z_{t-p-1}$ are the regressors (adding also a constant and a time trend):

$$z_{t-1} = \hat{\kappa}_0 + \hat{\kappa}_1 t + \sum_{i=1}^{p-1} \hat{\Xi}_i \triangle z_{t-i} + \hat{\nu}_t \tag{4.4.4}$$

with $\hat{\nu}_t$ being the $n \times 1$ vector of residual of the second set of regressions. The second step consists of computing the sample variance-covariance matrices of the OLS residuals $\hat{\eta}_t$ and $\hat{\nu}_t$:

$$\begin{aligned}
\hat{\Sigma}_{\eta\eta} &= \frac{1}{T} \sum_{t=1}^{T} \hat{\eta}_t \hat{\eta}_t' \\
\hat{\Sigma}_{\nu\nu} &= \frac{1}{T} \sum_{t=1}^{T} \hat{\nu}_t \hat{\nu}_t' \\
\hat{\Sigma}_{\eta\nu} &= \frac{1}{T} \sum_{t=1}^{T} \hat{\eta}_t \hat{\nu}_t' \\
\hat{\Sigma}_{\nu\eta} &= \hat{\Sigma}_{\eta\nu}'
\end{aligned}$$

and from these computing the eigenvalues λ_i of the matrix:

$$\hat{\Sigma}_{\nu\nu}^{-1} \hat{\Sigma}_{\nu\eta} \hat{\Sigma}_{\eta\eta}^{-1} \hat{\Sigma}_{\eta\nu}. \tag{4.4.5}$$

Sorting the eigenvalues so that $\hat{\lambda}_1 > \hat{\lambda}_2 > \ldots > \hat{\lambda}_n$, the maximum value of the loglikelihood function subject to the constraint that there are h cointegration relationships is given by:

$$L_0^* = -\frac{Tn}{2}\log(2\pi) - \frac{Tn}{2} - \frac{T}{2}\log\mid\hat{\Sigma}_{\eta\eta}\mid - \frac{T}{2}\sum_{i=1}^{h}\log(1-\hat{\lambda}_i). \tag{4.4.6}$$

If we consider L_0^* as the constrained loglikelihood function, we can test the hypothesis that there are exactly h cointegrating relationships using the likelihood ratio test statistic. Let H_1 be the alternative hypothesis that there are exactly $h+1$ cointegration relationships, following the same reasoning as above, the maximized loglikelihood will be:

$$L_1^* = -\frac{Tn}{2}\log(2\pi) - \frac{Tn}{2} - \frac{T}{2}\log\mid\hat{\Sigma}_{\eta\eta}\mid - \frac{T}{2}\sum_{i=1}^{h+1}\log(1-\hat{\lambda}_i) \tag{4.4.7}$$

and the loglikelihood ratio test of H_0 (h cointegration relationships) against H_1 is:

$$\begin{aligned} L_1^* - L_0^* &= -\frac{T}{2}\sum_{i=1}^{h+1}\log(1-\hat{\lambda}_i) - \frac{T}{2}\sum_{i=1}^{h}\log(1-\hat{\lambda}_i) \\ &= -\frac{T}{2}\log(1-\hat{\lambda}_{h+1}) \end{aligned}$$

or also:

$$2(L_1^* - L_0^*) = -T\log(1-\hat{\lambda}_{h+1}). \tag{4.4.8}$$

This is the maximum eigenvalue test statistic of the Johansen cointegration test and its distribution is given in Johansen (1998 and 1991).

The testing procedure will be the following: we start by testing the null hypothesis $H_0 : h = 0$ against the unilateral alternative $H_1 : h \geq 1$. If the null hypothesis is rejected we will compute the test $H_0 : h = 1$ against $H_1 : h \geq 2$ and so on iteratively until when the null is not rejected. The value h for which H_0 is not rejected will be the number of cointegration relationships of our vector of variables. We mentioned before that if $h = n$ (i.e. the series are all cointegrated and $\Theta(1)$ has full rank), then the vector z_t is jointly stationary.

When there is no evidence of joint stationarity of the vector z_t, it could be interesting to investigate which one of the n variables is the source of non-stationarity. This will be done using the augmented Dickey-Fuller test. Let z_{it} be the i-th element of z_t. We assume that z_{it} is generated by an autoregressive process of order p:

$$\phi(L)z_{it} = \alpha + \delta t + \varepsilon_{it}$$

with $\phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p$. Applying to $\phi(L)$ the lemma of Section 2.3.1, we can rewrite the $AR(p)$ process as:

$$\begin{aligned}
[\phi(1)L + \phi^*(L)(1-L)]\, z_{it} &= \alpha + \delta t + \varepsilon_{it} \\
\triangle z_{it} &= \alpha + \delta t - \phi(1) z_{i,t-1} - \sum_{j=1}^{p-1} \phi_j^* \triangle z_{i,t-j} + \varepsilon_{it} \quad \text{or also:} \\
z_{it} &= \alpha + \delta t + \rho z_{i,t-1} - \sum_{j=1}^{p-1} \phi_j^* \triangle z_{i,t-j} + \varepsilon_{it} \qquad (4.4.9)
\end{aligned}$$

with $\rho = 1 - \phi(1)$. If z_{it} contains a unit root, i.e. is integrated of order one, $\sum_j \phi_j^* \triangle z_{i,t-j}$ will be integrated of order 0 and is, thus, stationary. It follows that z_{it} is non-stationary only if $\rho z_{i,t-1}$ is non-stationary. Conversely, z_{it} is stationary if $\rho z_{i,t-1}$ is stationary, which is the case when $\rho < 1$. In practice, we will test the null hypothesis $H_0 : \rho = 1$ against the unilateral alternative $H_1 : \rho < 1$ and the test statistic (augmented Dickey-Fuller's test) will be:

$$t_{adf} = \frac{\hat{\rho} - 1}{\hat{\sigma}_{\hat{\rho}}}$$

where $\hat{\rho}$ is the ordinary least squares estimation of ρ and $\sigma^2_{\hat{\rho}}$ its estimated variance. The distribution of the augmented Dickey-Fuller test statistic is non-standard because, under the null, ρ is the coefficient of an $I(1)$ variable. The critical values can be computed by simulation and can be found, for example, in Hamilton (1994).

In summary, the strategy we will employ to test whether or not the vector of variables presented in Section 4.3 is stationary will be firstly, to compute Johansen's cointegration test to check if the series are jointly stationary. Secondly, if there is no evidence of joint stationarity, we will compute the augmented Dickey-Fuller test on the individual variables in order to discover which one contains unit roots.

4.5 Results

4.5.1 Graphical Representation

Before starting to compute the stationarity tests we presented, it may be interesting to have a graphical overview of the series (detailed descriptive statistics are also available in Appendix B). In Figures 4.6 to 4.11, seasonally adjusted and unadjusted data are plotted, in levels, first-differences and seasonal differences (cf. Table 4.2).

At first sight it is difficult to say whether the series are stationary or not, although we can unveil some distinctive patterns. Budget shares seem to follow cyclical patterns with high frequency (short term) fluctuations, probably due to seasonal factors. Nevertheless, over the whole period their level does not exhibit substantial variations; we can recognize a slight growth in Italian and US shares, and a probable decrease in German ones. Seasonality clearly appears in Italian budget shares, with regular peaks and spikes during summer months, while, during the rest of the year they stay quite constant.

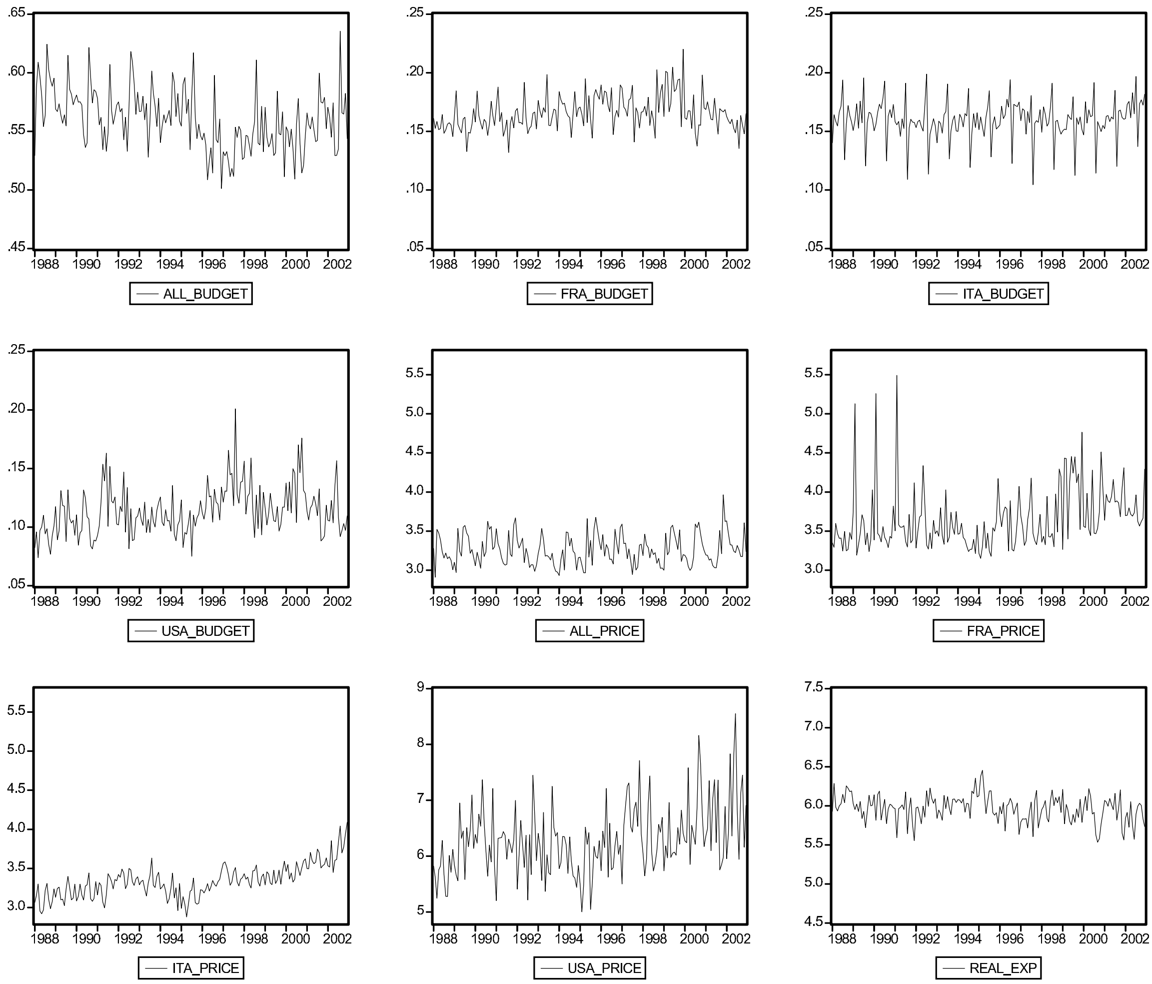

Figure 4.6: Seasonally unadjusted budget shares, import prices and real expenditure (levels, 1988-2002)

The impact of seasonality can be clearly seen by comparing Figures 4.6 and 4.7. Seasonal adjustment consistently reduces the fluctuations for all the budget shares series, however other patterns remain (and probably some residual seasonality). Seasonal adjustment has, thus, the effect of eliminating short term fluctuations that may cause non-stationarity, but it cannot solve unit roots problems, if they exist.

Concerning prices, we can see that German import prices seem to be cyclical but stationary over the period, while French, Italian and US prices grow, although

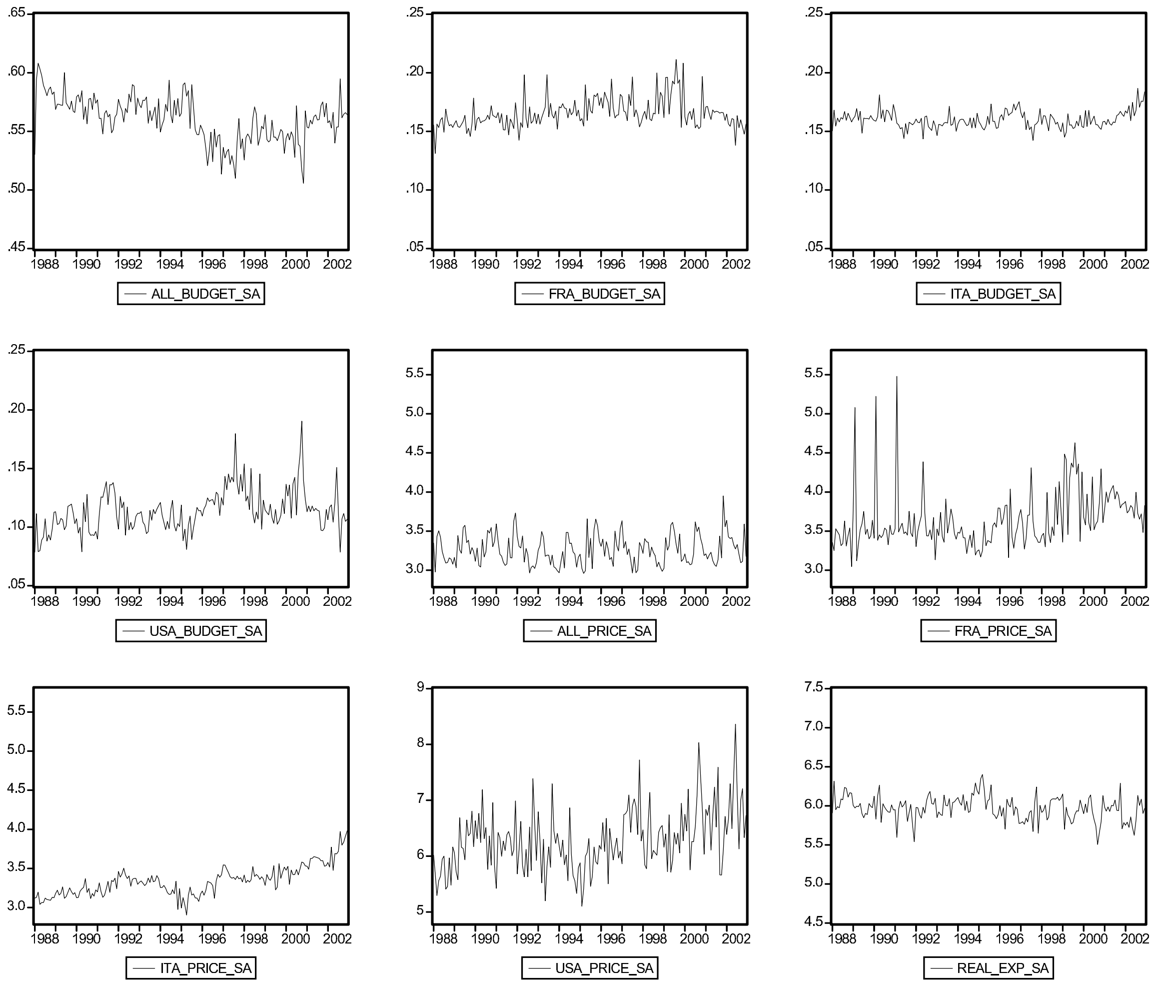

Figure 4.7: Seasonally adjusted budget shares, import prices and real expenditure (levels, 1988-2002)

with different patterns. The French series reveals the presence of outliers probably due to particular transactions we did not investigate (the outliers are the observations corresponding to February 1989, 1990 and 1991). Italian and US prices grew regularly over the period but with respectively small and large fluctuations. The differences between countries are probably due to the composition of the basket of imported goods and the evolution of exchange rates. On the other hand, the real expenditure term seems to be stationary (it slightly decreases) and does not show irregular variations

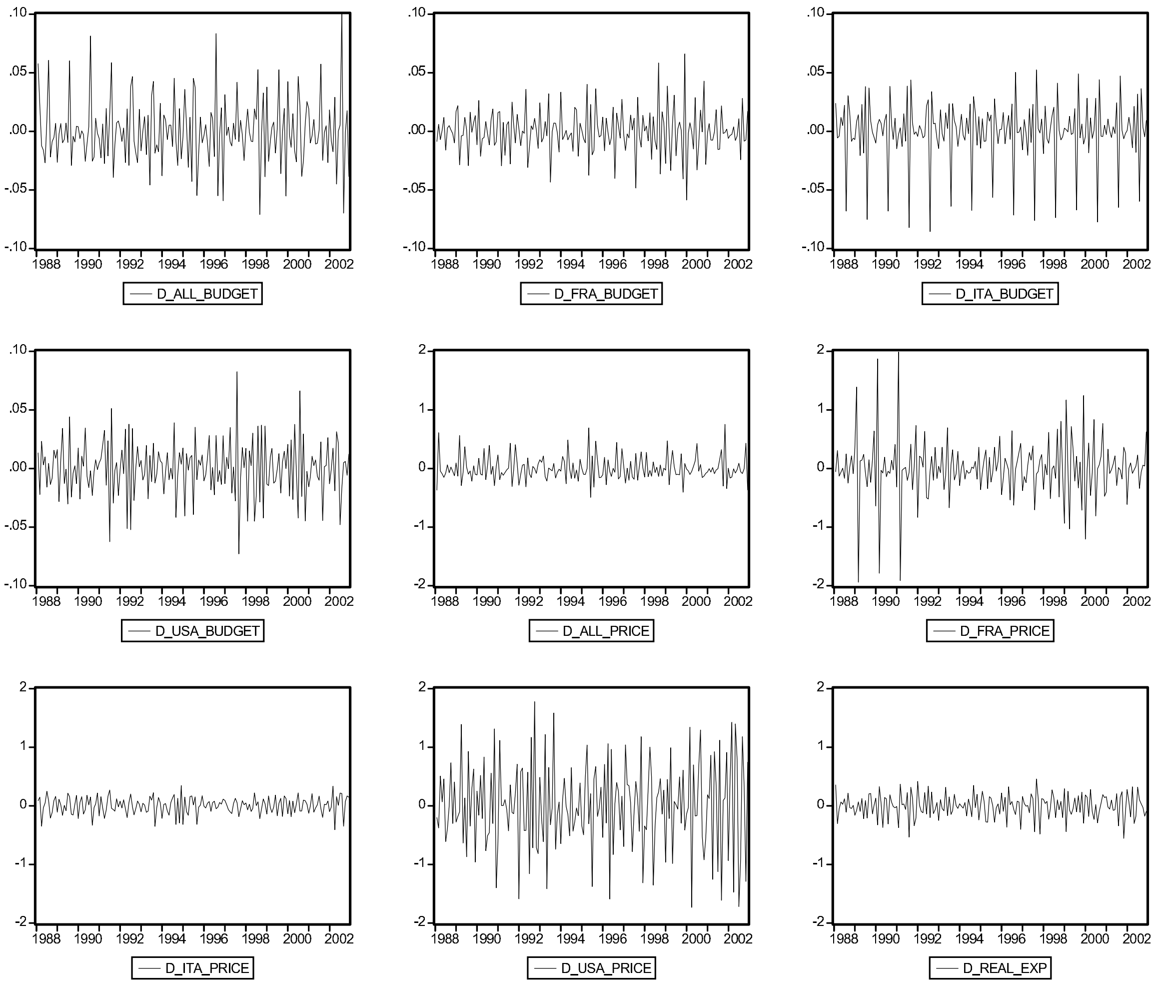

Figure 4.8: Seasonally unadjusted budget shares, import prices and real expenditure (first-differences, 1988-2002)

or a particular pattern.

Seasonal effects seem to be smaller than for budget shares. Seasonal adjustment has a limited impact – fluctuations are only slightly reduced. This is, in some ways, expected because prices are stickier and do not depend on seasonal factors.

First-differences are plotted in Figures 4.8 and 4.9. Short term fluctuations and the effect of seasonal adjustment appear even more clearly than before, especially for budget shares and US prices. The range of fluctuation is strongly reduced, the

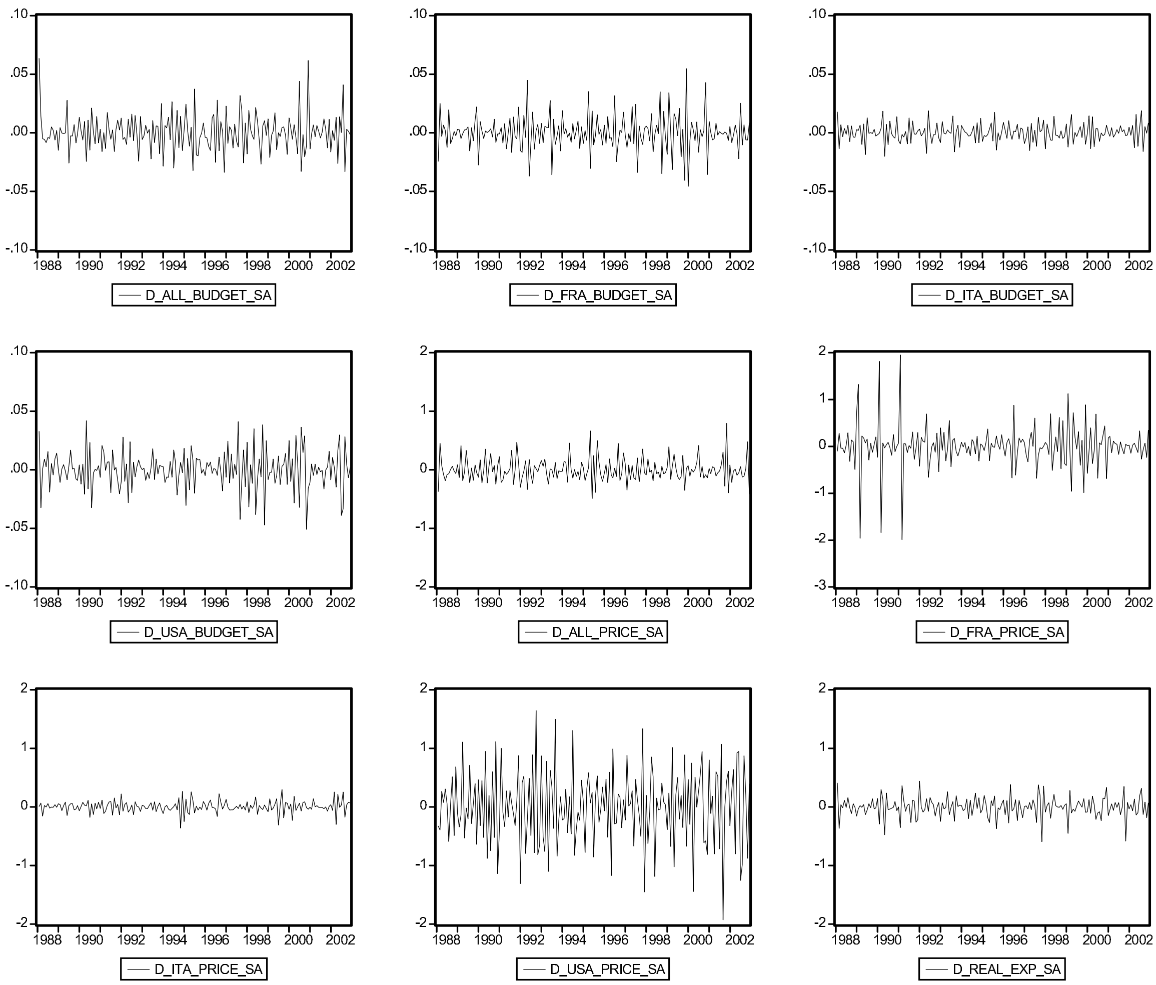

Figure 4.9: Seasonally adjusted budget shares, import prices and real expenditure (first-differences, 1988-2002)

flagrant example of Italian budget shares illustrated this effect.

It is evident that seasonality is a major source of non-stationarity in level and first-differences data, but it seems not to be the only one. In addition, first-differences (adjusted and unadjusted) seem to represent only short-term fluctuations with noticeable amplitude. Fluctuations are probably still due to seasonal factors and are, thus, of little interest to our study.

Figures 4.10 and 4.11, where seasonal differences are plotted, lead to different

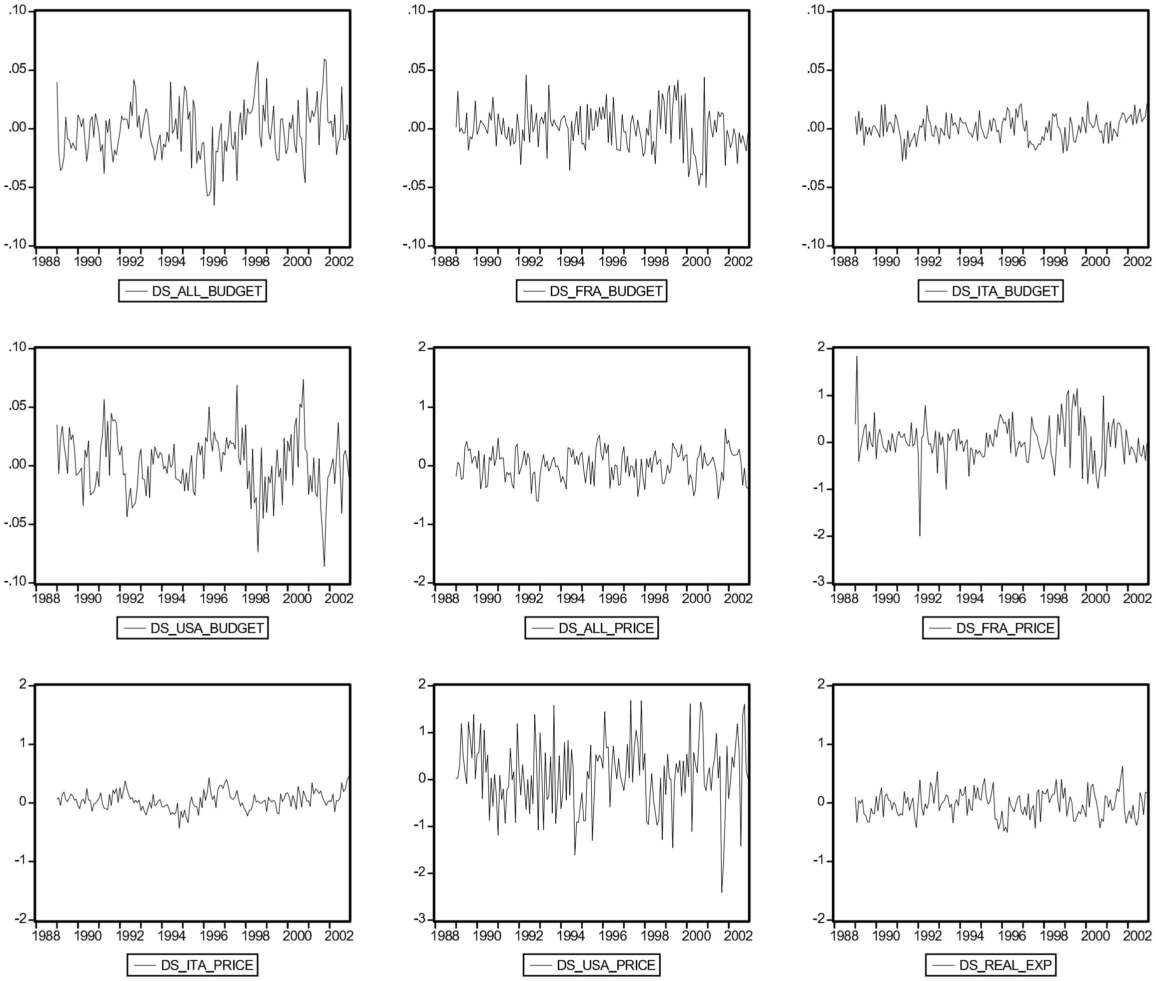

Figure 4.10: Seasonally unadjusted budget shares, import prices and real expenditure (seasonal differences, 1988-2002)

conclusions. The series are substantially less noisy and seem to be more able to capture long-term patterns. Seasonal differences have the advantage of eliminating seasonal roots (if these are present) and correcting seasonality effects. It is, thus, not surprising that unadjusted and adjusted data are very similar (in Figure 4.5 there is already no evidence of seasonality in unadjusted data, all the monthly means are around zero). But again, noticeable fluctuations remain and they are not due to seasonal factors.

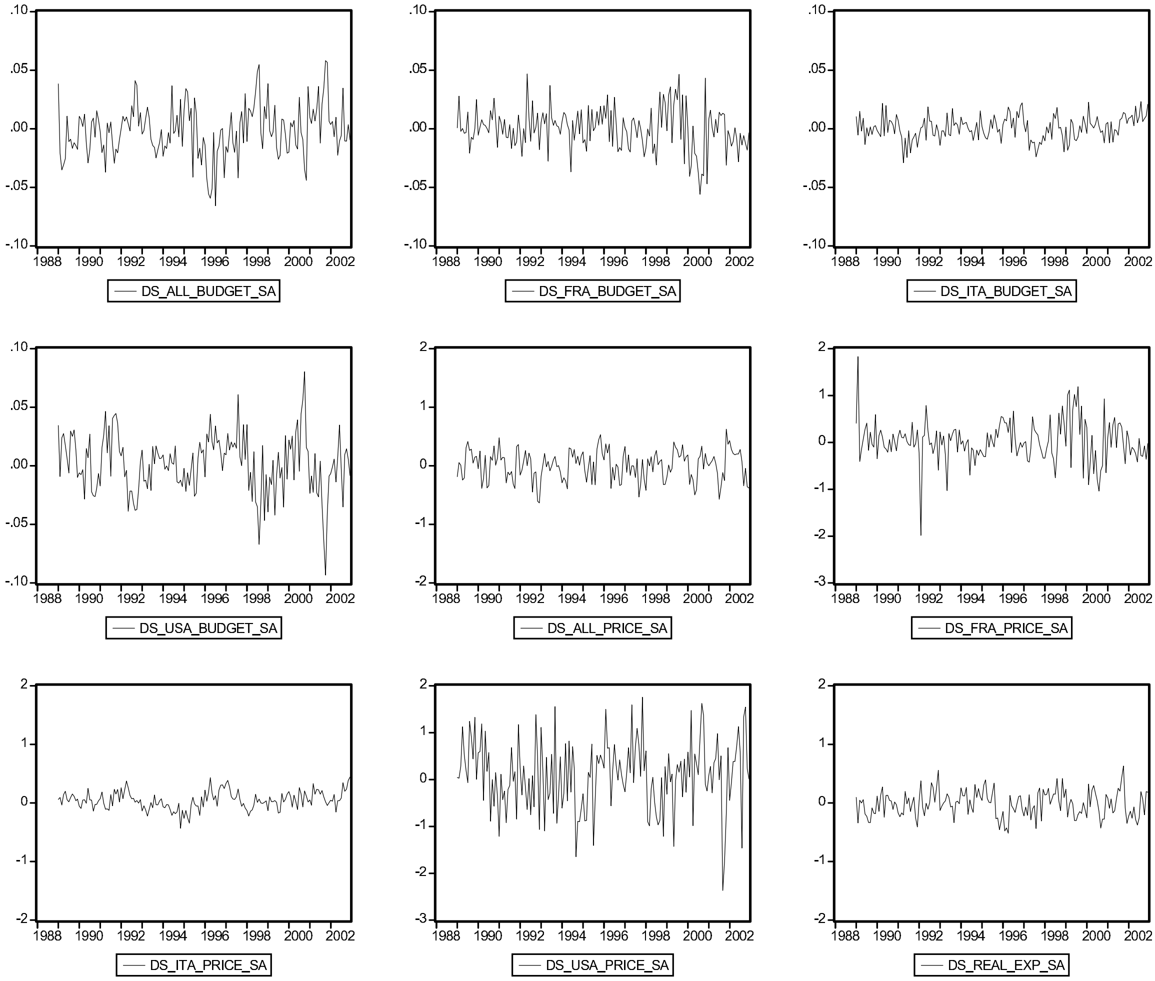

Figure 4.11: Seasonally adjusted budget shares, import prices and real expenditure (seasonal differences), 1988-2002)

As a preliminary conclusion, we can then affirm that a cyclical or regular pattern can be recognized in part of the series (at least levels and seasonal differences), but it is difficult to say whether or not the series are stationary. Seasonality is a factor of variability, but, in fact, seasonal adjustment has the benefit of reducing part of the short term fluctuations (for levels and first-differences data, especially in budget shares). Although, if the data are non-stationary, our first impression is that seasonal adjustment will not be enough to eliminate unit roots.

4.5.2 Stationarity Tests and Data Specification

The next step of our analysis is to compute stationarity tests in order to have more rigorous results after this graphical overview. In Section 4.2 we explained that stationarity is a crucial requirement for the identification of the equilibrium multipliers of the general dynamic model $\Phi(L)w_t = \Gamma(L)x_t + u_t$. The two basic assumptions we made are that the processes generating the vector of endogenous and exogenous variables (w_t and x_t) are covariance-stationary, and in Section 4.4, we showed that these hypotheses can be tested with the help of Johansen's cointegration test. We will focus first on the vector of endogenous variables $w_t = \begin{pmatrix} w_t^{Ger} & w_t^{Fra} & w_t^{Ita} \end{pmatrix}'$.[3] Assuming that w_t is generated by an autoregressive process of the form $\Phi(L)w_t = \mu + \delta t + u_t$, with $\Phi(L) = I - \Phi_1 L - \ldots - \Phi_p L^p$, we showed that this process can be rewritten as:

$$\triangle w_t = \mu + \delta t + \Phi(1)w_{t-1} - \sum_{j=1}^{p-1} \Phi_j^* \triangle w_{t-j} + u_t \qquad (4.5.1)$$

where $\Phi_j^* = -\sum_{s=j+1}^{p} \Phi_s$ and $\Phi_0^* = \Phi_0 = I$. Johansen's cointegration test will help us to determine the number of cointegration relationships between the variables composing the vector w_t, and, in this way, determine the rank of the matrix $\Phi(1)$. Remember that w_t (or w_{t-1} in equation (4.5.1)) is (jointly) stationary if $\Phi(1)$ is of full rank, i.e. if three cointegration relationships exist.

Table 4.4 summarizes the cointegration test results using the six data specifications presented in Section 4.3 (unadjusted and seasonally adjusted levels, first-differences and seasonal differences). For all the specifications except seasonally adjusted levels (w_t^{sa}), Johansen's test reveals the presence of three cointegration relationships at both 5% and 1%. We may then conclude that, for w_t, $\triangle w_t$, $\triangle w_t^{sa}$, $\triangle_s w_t$ and $\triangle_s w_t^{sa}$, the

[3] Since the budget shares have to sum to unity, the last equation is redundant and is deleted (see Section 2.4.1). As for estimation, we consider only the incomplete system.

H_0	H_1	Maximum-eigenvalues statistic						Critical values	
		w_t	w_t^{sa}	$\triangle w_t$	$\triangle w_t^{sa}$	$\triangle_s w_t$	$\triangle_s w_t^{sa}$	5%	1%
$h = 0$	$h \geq 1$	47.68	26.01	91.64	107.05	29.96	30.94	20.97	25.52
$h \leq 1$	$h \geq 2$	20.91	12.33	64.10	98.10	22.75	22.08	14.07	18.63
$h \leq 2$	$h = 3$	11.83	8.24	40.36	87.30	15.45	14.91	3.76	6.65
Lag length†	p	3	2§	5§	2	2	2		

† Lag length refers to the smallest number of lags in equation (4.5.1) for which the null hypothesis of no serial correlation in the LM test is not rejected at 5% (we interpreted the presence of serial correlation as a signal of misspecification)

§ The null hypothesis of no serial correlation is not rejected at 1%

Table 4.4: Johansen's cointegration test on budget shares

matrix $\Phi(1)$ has full rank and thus, the vectors are jointly stationary.

The result of Johansen's test for seasonally adjusted levels is contradictory. The null hypothesis cannot be rejected when we test $H_0 : h \leq 1$ against $H_1 : h \geq 2$, but it is rejected when we test $H_0 : h \leq 2$ against $H_1 : h = 3$. No conclusion can be drawn concerning the joint stationarity of the vector, but if we investigate the stationarity in individual variables (by computing augmented Dickey-Fuller's tests), we discover that the null hypothesis (the presence of unit roots) cannot be rejected for German budget shares (cf. Table 4.5).

Variable	t-statistic	5% critical value	1% critical value
w_{ger}^{sa}	-2.25		
w_{fra}^{sa}	-4.32	-3.44	-4.01
w_{ita}^{sa}	-4.28		

Table 4.5: Augmented Dickey-Fuller test on seasonally adjusted budget shares (levels)

The same procedure is applied to the vector of exogenous series, i.e. import prices and real expenditure. We saw that the stationarity of this vector is a crucial assumption for the stationarity of the general dynamic model. So, let:

$$x_t = \begin{pmatrix} p_t^{Ger} & p_t^{Fra} & p_t^{Ita} & p_t^{Usa} & \ln M_t/P_t \end{pmatrix}'$$

be the vector of import prices and real expenditure, and assume that it is generated by a VAR process of the form $\Theta(L)x_t = \mu + \delta t + u_t$ with $\Theta(L) = I - \Theta_1 L - \ldots - \Theta_q L^q$. We can rewrite this process as:

$$\triangle x_t = \mu + \delta t + \Theta(1)x_{t-1} - \sum_{j=1}^{q-1} \Theta_j^* \triangle x_{t-j} + u_t \tag{4.5.2}$$

with $\Theta_j^* = -\sum_{s=j+1}^{q} \Theta_s$ ($\Theta_0^* = \Theta_0 = I$) and where all the terms, including $\Theta(1)x_{t-1}$, are stationary. Again, Johansen's cointegration test will tell us how many cointegration relations exist between the variables composing x_t, and thus, whether or not x_t is jointly stationary.

H_0	H_1	**Maximum-eigenvalues statistic** x_t	x_t^{sa}	$\triangle x_t$	$\triangle x_t^{sa}$	$\triangle_s x_t$	$\triangle_s x_t^{sa}$	**Critical values** 5%	1%
$h = 0$	$h \geq 1$	49.81	38.89	75.68	134.24	65.00	64.29	33.46	38.77
$h \leq 1$	$h \geq 2$	39.05	32.08	58.88	112.88	54.32	53.81	27.07	32.24
$h \leq 2$	$h \geq 3$	33.10	25.30	55.93	99.98	42.55	41.59	20.97	25.52
$h \leq 3$	$h \geq 4$	15.60	14.18	45.04	77.79	24.21	25.61	14.07	18.63
$h \leq 4$	$h = 5$	0.02	0.02	34.18	73.05	15.92	16.12	3.76	6.65
Lag length	q	3	3	6	2	1	1		

Table 4.6: Johansen's cointegration test on the vector of exogenous variables

The hypothesis of five cointegration relationships is not rejected for all data specifications except seasonally adjusted and unadjusted levels. It means that we have evidence of joint stationarity for first and seasonal differences, but not for levels – no matter if the data are adjusted or not (seasonal adjustment has the effect of increasing the test statistic but it does not change the test result). The non-stationarity of levels is due to German and Italian prices, for which we can not reject the hypothesis of unit roots in individual stationarity tests (see Table 4.7).

Variable	**t-statistic**	**5% critical value**	**1% critical value**
p_{ger}	-1.97		
p_{fra}	-11.90		
p_{ita}	-1.33	-3.44	-4.01
p_{usa}	-11.00		
$\log M_t/P_t$	-10.15		
p^{sa}_{ger}	-1.80		
p^{sa}_{fra}	-11.79		
p^{sa}_{ita}	-1.78	-3.44	-4.01
p^{sa}_{usa}	-10.68		
$\log M^{sa}_t/P^{sa}_t$	-9.23		

Table 4.7: Augmented Dickey-Fuller test on import prices and real expenditure (seasonally adjusted and unadjusted levels)

Table 4.8 summarizes the stationarity test results for the vector of endogenous (budget shares) and exogenous variables (import prices and real expenditure) and for all data specifications.[4] According to the results of Johansen's cointegration test, for the sequel of our analysis we can choose between four data specifications: seasonally adjusted and unadjusted first-differences and seasonally adjusted and unadjusted seasonal differences. All of these specifications satisfy the conditions under which the general dynamic model is stationary and thus, the equilibrium multipliers are identified. Between them, we will choose unadjusted seasonal differences because they should keep more information and because they lead to more parsimonious models (their explanatory power seems to be higher compared to first-differences).

[4]The checkmark "✓" denotes the joint stationarity of the vector.

	Seasonally unadjusted		Seasonally adjusted	
	Budget shares	Prices and real exp	Budget shares	Prices and real exp
Levels	✓			
First-differences	✓	✓	✓	✓
Seasonal-differences	✓	✓	✓	✓

Table 4.8: Data specification and joint stationarity

4.5.3 Lag Specification

Once the data specification is chosen, we can determine the lag length of the polynomials $\Phi(L)$ and $\Gamma(L)$ in the general dynamic model:

$$\Phi(L)\triangle_s w_t = \Gamma(L)\triangle_s x_t + u_t \tag{4.5.3}$$

with $\Phi(L) = I - \Phi_1 L - \ldots - \Phi_p L^p$ and $\Gamma(L) = \Gamma_0 + \Gamma_1 L + \ldots + \Gamma_q L^q$. We will estimate the model for different p's and q's by ordinary least squares, and determine the "best" lag specification according to the results of diagnostic and signification tests.

Table 4.9 summarizes the diagnostic test results on the OLS residuals for $p = 0, 1, 2, 3$ and $q = 0, 1, 2, 3$.[5] The first column refers to the multivariate extensions of the Jarque-Bera normality test (the null hypothesis is that the residuals are multivariate normal), the second to the multivariate LM test statistic for serial correlation up to the first order (under the null hypothesis the residuals are not correlated) and the third to the White heteroskedasticity test (residuals are heteroskedastic under the null).

[5] With monthly data we decided to not consider higher lags.

		Normality	No serial correlation	Heteroskedasticity
$p = 0$	$q = 0$	✓✓		✓✓
	$q = 1$	✓		✓✓
	$q = 2$	✓		✓✓
	$q = 3$	✓		✓✓
$p = 1$	$q = 0$	✓	✓✓	✓✓
	$q = 1$	✓		✓✓
	$q = 2$			✓✓
	$q = 3$			✓✓
$p = 2$	$q = 0$	✓	✓✓	✓✓
	$q = 1$	✓		✓✓
	$q = 2$	✓		✓✓
	$q = 3$		✓✓	✓✓
$p = 3$	$q = 0$	✓✓	✓	✓✓
	$q = 1$	✓✓	✓✓	✓✓
	$q = 2$	✓	✓✓	
	$q = 3$		✓✓	

Table 4.9: Diagnostic tests on the OLS residuals of the general dynamic model

As it can be seen in Table 4.9,[6] there is no evidence of departure from normality or homoscedasticity for almost all the different model specifications. However, for only a few of them the hypothesis of no-serial correlation in the residuals can not be rejected at a 5% critical level (for a larger number of models it is not rejected at a 1% level). We interpreted the presence of serial correlation as the consequence of mis-specification, this is why we restrict the choice of the lag length only to the models with non-autocorrelated errors. Between these, we first test the joint significance of the lagged budget shares ($\triangle_s w_{t-i}$, with $i = 2, 3$)[7] using a likelihood ratio test statistic corrected with the Anderson small-sample factor. For example, in order to test the significance of $\triangle_s w_{t-3}$ (i.e. the test of $p = 3$ against $p = 2$), we rewrite the

[6]The checkmarks"✓" and "✓✓" denote the non-rejection of the null hypothesis at respectively 1% and 5% for the first two columns, and rejection, at respectively 1% and 5%, of the null for the last column.

[7]The absence of lags is not considered because all the models with $p = 0$ have serially correlated errors.

unrestricted model (with $p = 3$) as:

$$\begin{aligned}\triangle_s w_t &= \Phi_3 \triangle_s w_{t-3} + \Phi_2 \triangle_s w_{t-2} + \Phi_1 \triangle_s w_{t-1} + \sum_{i=1}^{q} \Gamma_i \triangle_s x_{t-i} + u_t \\ &= \underbrace{\begin{pmatrix} \Phi_3 & \Phi_2 & \Phi_1 & \Gamma_0 & \dots & \Gamma_q \end{pmatrix}}_{\Upsilon} \triangle_s \mathcal{X}_t + u_t \end{aligned} \tag{4.5.4}$$

with $\triangle_s \mathcal{X}_t = \begin{pmatrix} \triangle_s w'_{t-3} & \triangle_s w'_{t-2} & \triangle_s w'_{t-1} & \triangle_s x'_t & \dots & \triangle_s x'_{t-q} \end{pmatrix}'$. If we define $\Upsilon = \begin{pmatrix} \Upsilon^* & \Upsilon^{**} \end{pmatrix}$ with $\Upsilon^* = \Phi_3$, we get the restricted model (with $p = 2$) when $\Upsilon^* = 0$. We then test the null hypothesis $H_0 : \Upsilon^* = 0$ against the alternative $H_1 : \Upsilon^* \neq 0$ using a LR test statistic $-2 \log \lambda = -2(L_{H_0} - L_{H_1})$ where L_{H_0} and L_{H_1} are the loglikelihoods of the restricted and unrestricted models. This statistic is corrected with the Anderson (1958) small-sample correction factor:

$$\tau = \frac{T - q_2 - \frac{1}{2}(g + q_1 + 1)}{T}$$

where q_1 is the number of columns of Υ^*, $q_2 = k - q_1$, k is the number of explanatory variables of the model and g is the number of rows of w_t. The corrected LR statistic $-2\tau \log \lambda$ follows a $\chi^2_{gq_1}$ distribution.[8] The same procedure can be applied to test the signification of $\triangle_s w_{t-2}$ (i.e. the test of $p = 1$ against $p = 2$, with $q = 0, 1, 2, 3$). The results are shown in Table 4.10.

For all values of q, at both 5% and 1% critical levels, we do not reject H_0 (i.e. the hypothesis that Υ^* is not significant) in the first set of tests. It clearly appears that $\triangle_s w_{t-3}$ is not significant, but this is not the case for $\triangle_s w_{t-2}$, because in the second set of tests the null hypothesis is always rejected. From the results of Table 4.10, there is no doubt that the lag length of $\Phi(L)$ has to be two (i.e. $p = 2$).

[8]Kiviet (1986) showed that even if the correction is not strictly valid due to the presence of lagged endogenous variables in the model, the null distribution of the adjusted LR statistic can be expected to be closer to the χ^2 distribution than that of the unadjusted LR statistic.

H_0	H_1	**Adj. LR test statistic**	**5% critical value**	**1% critical value**
$p = 2$, $q = 0$	$p = 3$, $q = 0$	0.30		
$p = 2$, $q = 1$	$p = 3$, $q = 1$	3.30	16.92	21.67
$p = 2$, $q = 2$	$p = 3$, $q = 2$	7.08		
$p = 2$, $q = 3$	$p = 3$, $q = 3$	4.91		
$p = 1$, $q = 0$	$p = 2$, $q = 0$	34.95		
$p = 1$, $q = 1$	$p = 2$, $q = 1$	31.83	16.92	21.67
$p = 1$, $q = 2$	$p = 2$, $q = 2$	34.90		
$p = 1$, $q = 3$	$p = 2$, $q = 3$	35.81		

Table 4.10: Lag exclusion test for $\Phi(L)$

Given that $p = 2$, we still have to determine the lag length of the polynomial specification ($\Gamma(L)$) associated to the vector of exogenous variables. This will be done by testing alternative specifications of q ($q = 0, 1, 2, 3$) using again a LR "adjusted" test statistic. The results of Table 4.11 tell us that the null hypothesis can not be rejected in any of the three tests, which means that the relevant order for $\Gamma(L)$ will be equal to 0 (i.e. $q = 0$).

H_0	H_1	**Adj. LR test statistic**	**5% critical value**	**1% critical value**
$p = 2$, $q = 0$	$p = 2$, $q = 1$	9.38		
$p = 2$, $q = 1$	$p = 2$, $q = 2$	12.03	28.87	34.81
$p = 2$, $q = 2$	$p = 2$, $q = 3$	10.24		

Table 4.11: Lag exclusion test for $\Gamma(L)$

Our procedure suggests then, to choose $p = 2$ and $q = 0$. However, with this lag specification the general dynamic model reduces to the partial-adjustment model (see Section 2.3.4). Therefore, we will estimate only three models: the static, autoregressive-errors and partial-adjustment models. The choice of different values of

p and q will also have some implications on the dynamic specification tests (see Section 5.3.3). In fact, with $p \neq q$ it will not be possible to test the autoregressive-errors model against the error-correction or partial-adjustment models. Imposing $p = q$ seemed too restrictive to us. If we look at the diagnostic test results (Table 4.9), we notice that the models with $p = q$ have either serial correlation or heteroskedasticity problems.

4.5.4 Summary

In summary, in this chapter we explained the importance of stationarity for the equilibrium multipliers estimation. We introduced the concept of joint stationarity and presented a testing procedure based on Johansen's cointegration test that has been applied to various data specifications, between which we chose seasonal differences. Finally, we tested the signification of lagged vectors of endogenous and exogenous variables and we found that the general dynamic model reduces to the partial-adjustment one:

$$\triangle_s w_t - \Phi_1 \triangle_s w_{t-1} - \Phi_2 \triangle_s w_{t-2} = \Gamma_0 \triangle_s x_t + u_t \tag{4.5.5}$$

or, defining $y_t = \triangle_s w_t$ and $z_t = \triangle_s x_t$ in order to simplify the notation,

$$y_t - \Phi_1 y_{t-1} - \Phi_2 y_{t-2} = \Gamma_0 z_t + u_t \tag{4.5.6}$$

and the associated error-correction model is:

$$\Phi^*(L) \triangle y_t = -\Phi(1) \left[y_{t-1} - \Pi z_t \right] + u_t \tag{4.5.7}$$

with:

$$\Phi^*(L) = \sum_{j=0}^{2-1} \Phi_j^* L^j = I + \Phi_2 L$$

$$\Phi(1) = I - \Phi_1 - \Phi_2$$
$$\Pi = \Phi^{-1}(1)\Gamma_0 = (I - \Phi_1 - \Phi_2)^{-1}\Gamma_0$$

or, equivalently,

$$\triangle y_t = -\Phi(1)\left[y_{t-1} - \Pi z_t\right] - \Phi_2 \triangle y_{t-1} + u_t. \tag{4.5.8}$$

This is the model we will estimate, in two alternative dynamic specifications (static and autoregressive-errors) and under the regularity assumptions (homogeneity and symmetry). These models, for $p = 2$ and $q = 0$, can be easily retrieved from the general error-correction model (or partial-adjustment since $q = 0$) by imposing the restrictions discussed in Section 2.3. Let's first rewrite equation (4.5.7) in terms of the original variables $\triangle_s w_t$ and $\triangle_s x_t$:

$$\Phi^*(L)\triangle\triangle_s w_t = -\Phi(1)\left[\triangle_s w_{t-1} - \Pi\triangle_s x_t\right] + u_t. \tag{4.5.9}$$

It is easy to show that this one can be rewritten as:

$$\Phi(L)\triangle_s w_t = \Phi(1)\Pi\triangle_s x_t + u_t \tag{4.5.10}$$

with $\Pi = \Phi^{-1}(1)\Gamma_0$. By imposing the restrictions introduced in Section 2.3 on equation (4.5.9) or (4.5.10) we get the static and autoregressive-errors models.

Chapter 5

Empirical Results

5.1 Introduction

With the data described in Section 4.1, taken in seasonal differences ($\triangle_s w_t = w_t - w_{t-12}$), and with the polynomial specifications found in Section 4.5, we will estimate the general dynamic model:

$$\Phi(L)\triangle_s w_t = \Gamma(L)\triangle_s x_t + u_t, \tag{5.1.1}$$

with:

$$\Phi(L) = I - \Phi_1 L - \Phi_2 L^2 \qquad (p = 2)$$

$$\Gamma(L) = \Gamma_0 \qquad (q = 0)$$

and:

$$\triangle_s w_t = \begin{pmatrix} \triangle_s w_t^{all} \\ \triangle_s w_t^{fra} \\ \triangle_s w_t^{ita} \end{pmatrix}$$

$$\triangle_s x_t = \begin{pmatrix} \triangle_s \ln p_t^{all} \\ \triangle_s \ln p_t^{fra} \\ \triangle_s \ln p_t^{ita} \\ \triangle_s \ln p_t^{usa} \\ \triangle_s \ln \frac{M_t}{P_t} \end{pmatrix},$$

in its static (ST), autoregressive-errors (AR) and partial-adjustment (PA) forms. Table 5.1 summarizes the restrictions and the models that will be estimated using the maximum likelihood procedure presented in Section 3.2.

	Restrictions	Model
Static	$\Phi_j = 0, \Gamma_i = 0$ $i, j > 0$	$\triangle_s w_t = \Pi \triangle_s x_t + u_t$ with $\Pi = \Gamma_0$
AR errors	$\Gamma(L) = \Phi(L)\Gamma_0$	$\triangle_s w_t = \Gamma_0 \triangle_s x_t + v_t$ with $v_t = \Phi_1 v_{t-1} + \Phi_2 v_{t-2} + u_t$
Partial-adjustment	$\Gamma_i = 0 \quad i > 0$	$\Phi(L)\triangle_s w_t = \Phi(1)\Pi\triangle_s x_t + u_t$ with $\Pi = \Phi^{-1}(1)\Gamma_0$

Table 5.1: The models and their restrictions

The three models will be estimated in their unconstrained form, under homogeneity and under both homogeneity and symmetry. In Section 2.4 we showed that the homogeneous estimates can be obtained by rewriting the model in terms of relative prices and setting the coefficients of the reference price equal to zero. This procedure consists of estimating the model omitting the variable chosen as the reference price. In our case, relative prices are computed with respect to the US import price

$(\triangle_s \ln p_t^{usa})$. The (unconstrained) regressors matrix $\triangle_s x_t^h$ will have the form:

$$\triangle_s x_t^h = \begin{pmatrix} \triangle_s \ln \frac{p_t^{all}}{p_t^{usa}} \\ \triangle_s \ln \frac{p_t^{fra}}{p_t^{usa}} \\ \triangle_s \ln \frac{p_t^{ita}}{p_t^{usa}} \\ \triangle_s \ln p_t^{usa} \\ \triangle_s \ln \frac{M_t}{P_t} \end{pmatrix},$$

and the equilibrium multipliers matrix will be:

$$\Pi^h = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \sum_{j=1}^{n} \gamma_{1j} & \beta_1 \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \sum_{j=1}^{n} \gamma_{2j} & \beta_2 \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \sum_{j=1}^{n} \gamma_{3j} & \beta_3 \end{pmatrix}$$

where the γ_{ij} are the price coefficients and the β_i are the real expenditure term coefficients. Homogeneity implies that $\sum_{j=1}^{n} \gamma_{ij} = 0$ for each $i = 1, 2, 3$, it follows that the general dynamic homogeneous model (with $p = 2$ and $q = 0$) can be written as:

$$\Phi(L)\triangle_s w_t = \Pi^* \triangle_s x_t^* + u_t, \tag{5.1.2}$$

where:

$$\triangle_s x_t^* = \begin{pmatrix} \triangle_s \ln \frac{p_t^{all}}{p_t^{usa}} \\ \triangle_s \ln \frac{p_t^{fra}}{p_t^{usa}} \\ \triangle_s \ln \frac{p_t^{ita}}{p_t^{usa}} \\ \triangle_s \ln \frac{M_t}{P_t} \end{pmatrix},$$

and

$$\Pi^* = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \beta_1 \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \beta_2 \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \beta_3 \end{pmatrix}.$$

Symmetry implies that the cross-price effects between countries are symmetric, that is $\gamma_{ij} = \gamma_{ji}$ for $i \neq j$. Constrained estimates can be computed by rewriting the model in a way where only the lower (or upper) triangle of the price coefficients matrix appears (see Section 2.4). The starting point is the homogeneous model in vector error-correction (VEC) form:

$$\Phi(L)\triangle_s w_t = (\triangle_s x_t^{*\prime} \otimes I_3)\ \text{vec}\,\Pi^* + u_t.$$

By defining:

$$\pi_s = \begin{pmatrix}\gamma_{11} & \gamma_{21} & \gamma_{22} & \gamma_{31} & \gamma_{32} & \gamma_{33} & \beta_1 & \beta_2 & \beta_3\end{pmatrix}'$$

as the vector regrouping the lower triangle price coefficients of Π^* and the coefficients associated to the real expenditure term, and by defining D_s as a selection matrix such that $\text{vec}\,\Pi^* = D_s\pi_s$, the general dynamic symmetric and homogeneous model can be written as:

$$\Phi(L)\triangle_s w_t = (\triangle_s x_t^{*\prime} \otimes I_3)\, D_s\pi_s + u_t.$$

In the first part of the chapter, equilibrium multipliers, Slutsky coefficients and elasticities estimates will be presented for each of the dynamic specifications and under the different economic theory assumptions (homogeneity and symmetry). The models will be evaluated according to goodness-of-fit measures, information criteria and residual analysis. The second part of the chapter will focus on hypothesis testing: firstly, the assumptions derived from the economic theory (for each of the dynamic specifications) and secondly, the dynamic specifications (under the different regularity assumptions). A comparison of the results will close the chapter.

Our goals are thus, on one hand, to check whether or not we can reject the regularity restrictions imposed in demand theory and, on the other hand, to compare the model performances and check whether we can reject the static against the dynamic model specifications.

5.2 Estimation

5.2.1 Equilibrium Multipliers

Tables 5.3 to 5.5 present the equilibrium multipliers estimates for the three model specifications.[1] The figures indicate the coefficient estimates followed by their estimated asymptotic standard-errors and the t-statistic for the test $H_0 : \gamma_{ij} = 0$ (or $H_0 : \beta_i = 0$) against $H_1 : \gamma_{ij} \neq 0$ (or $H_1 : \beta_i \neq 0$). The estimates and their asymptotic standard-errors are computed using the maximum likelihood procedure presented in Section 3.2, while the coefficients and standard-errors of the last (and deleted) equation follow from the adding-up restrictions (see Section 2.4).[2]

These figures are not, by themselves, of great interest and in addition, part of the estimated coefficients are not individually significant. It is, however, interesting to notice that the dynamic specification does not produce large differences in the coefficient estimates (under the same economic hypotheses the results are similar for the static, autoregressive-errors and partial-adjustment models). In contrast, when imposing homogeneity, large differences (compared to the unconstrained models) arise. The introduction of the symmetry constraints has a minor impact. If we compare the results of Tables 5.4 and 5.5 we see that they are similar.

We may also notice a problem of plausibility concerning the sign of the β_i: because of the nature of the goods (manufactures)[3], we would expect positive coefficients but

[1] We used the Gauss 5.0 software to estimate all the models.

[2] The coefficients estimates of the US equation are obtained by multiplying the equilibrium multiplier matrix by $-i_3'$, where i_3' is a 1×3 row vector whose elements are equal to unity, standard-errors are computed consequently.

[3] Manufactures are defined here as a broad aggregate – sections 5-8 of the SITC classification – we do not expect then that they behave as complementary or inferior goods.

this is not the case in every equation, especially with homogeneous and symmetric estimation. The real expenditure term coefficients (β_i) for the US equation are all (significantly) negative with unconstrained estimation, and under homogeneity or symmetry, the number of coefficients with an inplausible sign is even greater (although, under homogeneity, the negative β_i for the French and Italian equations are not always significant).

	Economic (or regularity) restrictions		
	Unconstrained	Homogeneous	Symmetric and homogeneous
Static	15	12	9
Autoregressive-errors	33	30	27
Partial-adjustment	33	30	27

Table 5.2: The number of estimated coefficients according to the dynamic and economic restrictions

To what concerns the asymptotic standard-errors, the introduction of the regularity and/or dynamic restrictions has a similar impact. Homogeneity, by setting the last column of price coefficients equal to zero, and symmetry, by posing $\gamma_{ij} = \gamma_{ji}$ for $i \neq j$, have the effect of reducing the number of parameters to be estimated. This happens also when imposing the restrictions on the dynamic specification (Table 5.2 summarizes the number of estimated coefficients according to the dynamic specifications and under the different economic restrictions). As a result, the asymptotic standard-errors are reduced. We have then, on the one side, that the standard-errors of the symmetric models are, in general, the smallest, followed by the homogeneous and unconstrained models. On the other side, it is the static model that provides the smallest standard-errors, before the autoregressive-errors and partial-adjustment models.

	Static model					Autoregressive-errors model					Partial-adjustment model				
	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	β_i
Germany	0.099	0.007	0.007	0.015	0.173	0.102	0.008	0.010	0.016	0.174	0.101	0.004	0.002	0.015	0.180
	(0.010)	(0.003)	(0.007)	(0.003)	(0.015)	(0.010)	(0.003)	(0.008)	(0.003)	(0.015)	(0.016)	(0.006)	(0.012)	(0.004)	(0.026)
	9.90	2.33	1.00	5.00	11.53	10.20	2.67	1.25	5.33	11.60	6.31	0.67	0.17	3.75	6.92
France	0.014	0.039	-0.007	0.002	0.033	0.015	0.039	-0.009	0.003	0.033	0.020	0.041	-0.004	0.003	0.042
	(0.007)	(0.002)	(0.005)	(0.002)	(0.011)	(0.007)	(0.002)	(0.005)	(0.002)	(0.011)	(0.008)	(0.003)	(0.007)	(0.002)	(0.014)
	2.00	19.50	-1.40	1.00	3.00	2.14	19.50	-1.80	1.50	3.00	2.50	13.67	-0.57	1.50	3.00
Italy	0.021	-0.004	0.030	0.001	0.028	0.015	-0.006	0.027	0.001	0.021	0.036	-0.0001	0.044	0.002	0.054
	(0.006)	(0.002)	(0.004)	(0.002)	(0.009)	(0.006)	(0.002)	(0.005)	(0.002)	(0.009)	(0.011)	(0.004)	(0.008)	(0.003)	(0.018)
	3.50	-2.00	7.50	0.50	3.11	2.50	-3.00	5.40	0.50	2.33	3.27	-0.03	5.50	0.67	3.00
USA	-0.134	-0.042	-0.030	-0.018	-0.234	-0.132	-0.041	-0.028	-0.020	-0.228	-0.157	-0.045	-0.042	-0.020	-0.276
	(0.008)	(0.003)	(0.006)	(0.002)	(0.013)	(0.008)	(0.003)	(0.007)	(0.002)	(0.013)	(0.011)	(0.004)	(0.009)	(0.003)	(0.019)
	-16.75	-14.00	-5.00	-9.00	-18.00	-16.50	-13.67	-4.00	-10.00	-17.54	-14.27	-11.28	-4.67	-6.67	-14.53

Table 5.3: Equilibrium multipliers estimates, standard-errors and t-statistics (unconstrained static, AR and PA models)

	Static model				**Autoregressive-errors model**				**Partial-adjustment model**			
	γ_{i1}	γ_{i2}	γ_{i3}	β_i	γ_{i1}	γ_{i2}	γ_{i3}	β_i	γ_{i1}	γ_{i2}	γ_{i3}	β_i
Germany	0.035	-0.007	-0.028	0.078	0.037	-0.007	-0.030	0.074	0.040	-0.010	-0.029	0.092
	(0.0047)	(0.0029)	(0.0062)	(0.0081)	(0.0050)	(0.0027)	(0.0065)	(0.0085)	(0.0070)	(0.0055)	(0.0093)	(0.0122)
	7.45	-2.41	-4.52	9.63	7.40	-2.59	-4.62	8.71	5.71	-1.82	-3.12	7.54
France	-0.010	0.033	-0.020	-0.004	-0.007	0.033	-0.022	-0.003	-0.010	0.034	-0.019	-0.0004
	(0.0031)	(0.0019)	(0.0040)	(0.0053)	(0.0031)	(0.0018)	(0.0041)	(0.0052)	(0.0038)	(0.0030)	(0.0050)	(0.0066)
	-3.23	17.37	-5.00	-0.75	-2.26	18.33	-5.37	-0.58	-2.63	11.33	-3.80	-0.06
Italy	-0.003	-0.009	0.017	-0.009	-0.001	-0.010	0.015	-0.006	-0.005	-0.009	0.023	-0.005
	(0.0026)	(0.0016)	(0.0035)	(0.0045)	(0.0026)	(0.0014)	(0.0035)	(0.0045)	(0.0053)	(0.0041)	(0.0070)	(0.0092)
	-1.15	-5.63	4.86	-2.00	-0.38	-7.14	4.29	-1.33	-0.94	-2.20	3.29	-0.54
USA	-0.022	-0.017	0.031	-0.065	-0.029	-0.016	0.037	-0.065	-0.025	-0.015	0.025	-0.087
	(0.0054)	(0.0033)	(0.0071)	(0.0093)	(0.0054)	(0.0028)	(0.0072)	(0.0092)	(0.0090)	(0.0070)	(0.0120)	(0.0157)
	-4.07	-5.15	4.37	-6.99	-5.37	-5.71	5.14	-7.07	-2.78	-2.14	2.08	-5.54

Table 5.4: Equilibrium multipliers estimates, standard-errors and t-statistics (homogeneous static, AR and PA models)

	Static model				Autoregressive-errors model				Partial-adjustment model			
	γ_{i1}	γ_{i2}	γ_{i3}	β_i	γ_{i1}	γ_{i2}	γ_{i3}	β_i	γ_{i1}	γ_{i2}	γ_{i3}	β_i
Germany	0.027	-0.013	-0.011	0.064	0.022	-0.013	-0.005	0.049	0.032	-0.014	-0.012	0.083
	(0.0032)	(0.0018)	(0.0025)	(0.0065)	(0.0031)	(0.0018)	(0.0025)	(0.0064)	(0.0055)	(0.0029)	(0.0050)	(0.0109)
	8.44	-7.22	-4.40	9.85	7.10	-7.22	-2.00	7.66	5.82	-4.83	-2.40	7.61
France	-0.013	0.030	-0.012	-0.011	-0.013	0.030	-0.011	-0.014	-0.014	0.033	-0.012	-0.004
	(0.0018)	(0.0016)	(0.0014)	(0.0041)	(0.0018)	(0.0016)	(0.0013)	(0.0039)	(0.0029)	(0.0024)	(0.0029)	(0.0062)
	-7.22	18.75	-8.57	-2.68	-7.22	18.75	-8.46	-3.59	-4.83	13.75	-4.14	-0.65
Italy	-0.011	-0.012	0.029	-0.018	-0.005	-0.011	0.021	-0.011	-0.012	-0.012	0.037	-0.013
	(0.0025)	(0.0014)	(0.0033)	(0.0045)	(0.0025)	(0.0013)	(0.0033)	(0.0044)	(0.0050)	(0.0029)	(0.0065)	(0.0099)
	-4.40	-8.57	8.79	-4.00	-2.00	-8.46	6.36	-2.50	-2.40	-4.14	5.69	-1.31
USA	-0.003	-0.005	-0.006	-0.035	-0.004	-0.006	-0.005	-0.024	-0.006	-0.007	-0.013	-0.066
	(0.0017)	(0.0011)	(0.0012)	(0.0075)	(0.0016)	(0.0010)	(0.0010)	(0.0067)	(0.0031)	(0.0019)	(0.0032)	(0.0163)
	-1.76	-4.55	-5.00	-4.67	-2.50	-6.00	-5.00	-3.58	-1.94	-3.68	-4.06	-4.05

Table 5.5: Equilibrium multipliers estimates, standard-errors and t-statistics (homogeneous and symmetric static, AR and PA models)

Reduced variances do not necessarily mean an increase in the parameters significance. The t-statistics for the homogeneous models are smaller compared to the unconstrained models, while the consequences of symmetry vary between the equations (in general, compared to the homogeneous estimates, the t-statistics increase for the German, French and Italian equations, and decrease for the US equation).

5.2.2 Slutsky Matrix

Our analysis will be principally focused on the Slutsky (or substitution) matrix and the price and income elasticities that can be computed directly from the equilibrium multipliers estimates. Economic theory defines the typical element of the Slutsky matrix as:

$$s^*_{ij} = \frac{\partial q_i(p,x)}{\partial p_j} + \frac{\partial q_i(p,x)}{\partial x} q_j(p,x)$$

where, as defined in Chapter 2, p is the vector of import prices, q_i the quantity imported from country i and x the total expenditure on imports (the budget shares w_i are defined as $p_i q_i / x$) . This can be equivalently written as:

$$s^*_{ij} = \frac{\partial \log q_i(p,x)}{\partial \log p_j} \frac{q_i}{p_j} + \frac{\partial \log q_i(p,x)}{\partial \log x} \frac{q_i}{x} q_j,$$

i.e. as a function of the uncompensated price elasticities η^u_{ij}, obtained by differentiating the identity $\log q_i(p,x) = \log w_i - \log p_i + \log x$ with respect to $\log p_j$,

$$\begin{aligned} \eta^u_{ij} &= -\delta_{ij} + \frac{1}{w_i} \frac{\partial w_i}{\partial \log p_j} \\ &= -\delta_{ij} + \frac{1}{w_i} \left(\gamma_{ij} - \beta_i \frac{\partial \log P^*}{\partial \log p_j} \right) \\ &= -\delta_{ij} + \frac{1}{w_i} \left(\gamma_{ij} - \beta_i w_j \right), \end{aligned} \tag{5.2.1}$$

where w_i is given by equation (2.2.6), and the income elasticities ε_i:

$$\begin{aligned}\varepsilon_i &= \frac{\partial \log q_i(p,x)}{\partial \log x} = 1 + \frac{1}{w_i}\frac{\partial w_i}{\partial \log x} \\ &= 1 + \frac{\beta_i}{w_i} \end{aligned} \tag{5.2.2}$$

where γ_{ij} and β_i are the parameters of the AID model, $\log P^*$ is Stone's price index and δ_{ij} is the Kronecker delta (i.e $\delta_{ij} = 1$ for $i = j$ and 0 otherwise). Thus,

$$s^*_{ij} = \eta^u_{ij}\frac{q_i}{p_j} + \varepsilon_i \frac{q_i q_j}{x}.$$

In practice it is easier not to use s^*_{ij} but a linear transformation $s_{ij} = \frac{p_i p_j}{x} s^*_{ij}$ which has no impact on the sign of the eigenvalues of the Slutsky matrix:

$$\begin{aligned} s_{ij} &= \eta^u_{ij}\frac{p_i q_i}{x} + \varepsilon_i \frac{p_i q_i}{x}\frac{p_j q_j}{x} \\ &= \eta^u_{ij} w_i + \varepsilon_i w_i w_j. \end{aligned} \tag{5.2.3}$$

As suggested, for example in Bewley (1986) or Green et al. (1990), the elements of the Slutsky matrix of the AID model will then be computed as:

$$\begin{aligned} s_{ij} &= \left(-\delta_{ij} + \frac{1}{\bar{w}_i}(\gamma_{ij} - \beta_i \bar{w}_j)\right)\bar{w}_i + \left(1 + \frac{\beta_i}{\bar{w}_i}\right)\bar{w}_i \bar{w}_j \\ &= \gamma_{ij} + \bar{w}_i \bar{w}_j - \bar{w}_i \delta_{ij} \end{aligned} \tag{5.2.4}$$

where the average budget shares $\bar{w}_i$ (and $\bar{w}_j$) are taken as an approximation of w_i (and w_j). Their variances will be equal to those of the equilibrium multipliers since the average budget shares are constant:

$$V(s_{ij}) = V(\gamma_{ij} + \bar{w}_i \bar{w}_j - \bar{w}_i \delta_{ij}) = V(\gamma_{ij}).$$

Demand theory (see for example Mas-Colell et al., 1995) implies, under Walras' law, homogeneity of degree zero and the weak axiom of revealed preference, that the

Slutsky matrix is negative semidefinite. This in turn implies that the the own-price effects s_{ii} are negative (or null). We also expect the cross-price coefficients to be positive.

Tables 5.6 to 5.8 summarize the Slutsky matrix estimates for each of the models, the asymptotic standard-errors and the t-statistics for the significance tests. The last column (s_{i4}) of Tables 5.7 and 5.8 is computed as $-\sum_{j=1}^{3} s_{ij}$ for each equation (country) i and for all the models – the standard-errors are computed consequently.[4] In general, the own-price coefficients are (significantly) negative as expected. Nonetheless, the negative sign of the own-price Slutsky coefficients is a consequence of the negative semidefiniteness of the Slutsky matrix, but in general, the converse is not necessarily true. For this purpose, we computed the eigenvalues and we found that three of them are effectively negative (the fourth is null) for all the models.[5] Hence, the Slutsky matrix is negative semidefinite as suggested by demand theory – under all economic restrictions and dynamic specifications. Below we will analyze in more detail the particularities under each of the economic restrictions.

Unconstrained Estimation: The signs of the Slutsky matrix elements are as expected, except for the cross-price coefficients of the US equation that are negative instead of being positive (but one of them is not significant in the static and autoregressive-errors models). The results across the models are of similar magnitude and this is particulary true for the symmetric cross-price elements of the first three equations (Germany, France and Italy). Looking at

[4]It is easy to show that by rewriting the homogeneous and symmetric models in terms of absolute prices, the coefficient of the variable chosen as reference price ($\triangle_s p_t^{usa}$) is $-\sum_{j=1}^{3} \gamma_{ij}$. The associated Slutsky matrix element will be $s_{i4} = -\sum_{j=1}^{3} \gamma_{ij} + \bar{w}_i \bar{w}_4 - \delta_{i4} \bar{w}_i = -\sum_{j=1}^{3} s_{ij}$ for all i.

[5]If the eigenvalues of the Slutsky matrix S^* are complex numbers we check if the symmetric matrix $S^* + S^{*\prime}$ is negative semidefinite. This a necessary and sufficient condition for the negative semidefiniteness of S^*, see for example the mathematical appendix of Mas-Colell et al., 1995.

	Static model				AR model				PA model			
	s_{i1}	s_{i2}	s_{i3}	s_{i4}	s_{i1}	s_{i2}	s_{i3}	s_{i4}	s_{i1}	s_{i2}	s_{i3}	s_{i4}
Germany	-0.148	0.100	0.096	0.079	-0.144	0.101	0.100	0.080	-0.145	0.097	0.092	0.079
	(0.0097)	(0.0032)	(0.0072)	(0.0028)	(0.0097)	(0.0031)	(0.0077)	(0.0027)	(0.0160)	(0.0062)	(0.0123)	(0.0040)
	-15.26	31.25	13.33	28.21	-14.85	32.58	12.99	29.63	-9.06	15.65	7.48	19.75
France	0.107	-0.099	0.019	0.021	0.108	-0.100	0.018	0.022	0.113	-0.097	0.023	0.021
	(0.0069)	(0.0023)	(0.0052)	(0.0020)	(0.0069)	(0.0023)	(0.0053)	(0.0020)	(0.0084)	(0.0032)	(0.0065)	(0.0021)
	15.51	-43.04	3.65	10.50	15.65	-43.48	3.40	11.00	13.45	-30.31	3.54	10.00
Italy	0.111	0.022	-0.104	0.019	0.105	0.021	-0.108	0.019	0.126	0.026	-0.090	0.020
	(0.0059)	(0.0019)	(0.0044)	(0.0017)	(0.0057)	(0.0019)	(0.0047)	(0.0016)	(0.0110)	(0.0042)	(0.0085)	(0.0028)
	18.81	11.58	-23.64	11.18	18.42	11.05	-22.98	11.88	11.45	6.19	-10.59	7.14
USA	-0.071	-0.023	-0.012	-0.120	-0.068	-0.022	-0.010	-0.120	-0.094	-0.026	-0.024	-0.121
	(0.0083)	(0.0028)	(0.0062)	(0.0024)	(0.0081)	(0.0026)	(0.0067)	(0.0023)	(0.0115)	(0.0044)	(0.0089)	(0.0029)
	-8.55	-8.21	-1.94	-50.00	-8.40	-8.46	-1.49	-52.17	-8.17	-5.91	-2.70	-41.72

Table 5.6: Slutsky matrix estimates, their standard-errors and t-statistics of the unconstrained static, AR and PA models

	Static model				**AR model**				**PA model**			
	s_{i1}	s_{i2}	s_{i3}	s_{i4}	s_{i1}	s_{i2}	s_{i3}	s_{i4}	s_{i1}	s_{i2}	s_{i3}	s_{i4}
Germany	-0.211	0.085	0.062	0.064	-0.209	0.086	0.059	0.064	-0.206	0.083	0.061	0.062
	(0.0047)	(0.0029)	(0.0062)	(0.002)	(0.0050)	(0.0027)	(0.0065)	(0.0019)	(0.0070)	(0.0055)	(0.0093)	(0.0035)
	-44.89	29.31	10.00	32.00	-41.80	31.85	9.08	33.68	-29.43	15.09	6.56	17.71
France	0.083	-0.105	0.006	0.016	0.086	-0.105	0.004	0.015	0.083	-0.104	0.007	0.014
	(0.0031)	(0.0019)	(0.0040)	(0.0013)	(0.0031)	(0.0018)	(0.0041)	(0.0013)	(0.0038)	(0.0030)	(0.0050)	(0.0019)
	26.77	-55.26	1.50	12.31	27.74	-58.33	0.98	11.54	21.84	-34.67	1.40	7.37
Italy	0.087	0.017	-0.117	0.013	0.088	0.017	-0.119	0.014	0.085	0.017	-0.111	0.009
	(0.0026)	(0.0016)	(0.0035)	(0.0011)	(0.0026)	(0.0014)	(0.0035)	(0.0010)	(0.0053)	(0.0041)	(0.0070)	(0.0026)
	33.46	10.63	-33.43	11.82	33.85	12.14	-34.00	14.00	16.04	4.15	-15.86	3.46
USA	0.041	0.003	0.049	-0.093	0.035	0.002	0.056	-0.093	0.038	0.004	0.043	-0.085
	(0.0054)	(0.0033)	(0.0071)	(0.0023)	(0.0054)	(0.0028)	(0.0072)	(0.0019)	(0.0090)	(0.0070)	(0.0120)	(0.0044)
	7.59	0.91	6.90	-40.43	6.48	0.71	7.78	-48.95	4.22	0.57	3.58	-19.32

Table 5.7: Slutsky matrix estimates, their standard-errors and t-statistics of the homogeneous static, AR and PA models

	Static model				**AR model**				**PA model**			
	s_{i1}	s_{i2}	s_{i3}	s_{i4}	s_{i1}	s_{i2}	s_{i3}	s_{i4}	s_{i1}	s_{i2}	s_{i3}	s_{i4}
Germany	-0.220	0.080	0.079	0.061	-0.224	0.080	0.084	0.060	-0.214	0.079	0.077	0.058
	(0.0032)	(0.0018)	(0.0025)	(0.0017)	(0.0031)	(0.0018)	(0.0025)	(0.0016)	(0.0055)	(0.0029)	(0.0050)	(0.0031)
	-68.75	44.44	31.60	35.88	-72.26	44.44	33.60	37.50	-38.91	27.24	15.40	18.71
France	0.080	-0.108	0.014	0.014	0.080	-0.108	0.015	0.013	0.079	-0.105	0.014	0.012
	(0.0018)	(0.0016)	(0.0014)	(0.0011)	(0.0018)	(0.0016)	(0.0013)	(0.0010)	(0.0029)	(0.0024)	(0.0029)	(0.0019)
	44.44	-67.50	10.00	12.73	44.44	-67.50	11.54	13.00	27.24	-43.75	4.83	6.32
Italy	0.079	0.014	-0.105	0.012	0.084	0.015	-0.112	0.013	0.077	0.014	-0.098	0.007
	(0.0025)	(0.0014)	(0.0033)	(0.0012)	(0.0025)	(0.0013)	(0.0033)	(0.0010)	(0.0050)	(0.0029)	(0.0065)	(0.0032)
	31.60	10.00	-31.82	10.00	33.60	11.54	-33.94	13.00	15.40	4.83	-15.08	2.19
USA	0.061	0.014	0.012	-0.087	0.060	0.013	0.013	-0.086	0.058	0.012	0.007	-0.077
	(0.0017)	(0.0011)	(0.0012)	(0.0021)	(0.0016)	(0.0010)	(0.0010)	(0.0018)	(0.0031)	(0.0019)	(0.0032)	(0.0049)
	35.88	12.73	10.00	-41.43	37.50	13.00	13.00	-47.78	18.71	6.32	2.19	-15.71

Table 5.8: Slutsky matrix estimates, their standard-errors and t-statistics of the homogeneous and symmetric static, AR and PA models

these results, we would not be able to exclude the presence of symmetry.

Homogeneous Estimation: The results obtained are plausible, with significant negative own-price and positive cross-price coefficients. In general, the differences across the models are reduced compared to the unconstrained estimates. We notice, however, an increase in the number of non-significant coefficients. We may again observe a symmetry in the magnitude of results of the cross-price coefficients. This fact has even been reinforced with the introduction of homogeneity.

Homogeneous and Symmetric Estimation: The unconstrained and homogeneous estimates indicate the possible presence of symmetry. Under this set of restrictions, own-price and cross-price coefficients are close to the corresponding homogeneous estimates, but we see an increase in the number of significant coefficients due to the smaller asymptotic standard-errors. All the elements of the Slutsky matrix are (highly) significant under the different dynamic specifications. In general, we observe then, that the introduction of symmetry does not modify (or only slightly modifies) the price coefficients estimates but it strongly reduces their variances. It is not, however, sufficient to say whether or not this set of restrictions is justified, the tests of the following sections will give us more information.

5.2.3 Price and Expenditure Elasticities

Price elasticities can be computed directly from the Slutsky coefficients. We showed before that uncompensated (or Marshallian) price elasticities can be computed as:

$$\eta_{ij}^{u} = -\delta_{ij} + \frac{1}{w_i}\left(\gamma_{ij} - \beta_i w_j\right),$$

and, because of the Slutsky equation, we have that the compensated (or Hicksian) elasticities are equal to:

$$\begin{aligned}\eta_{ij} &= \eta_{ij}^{u} + \varepsilon_i w_j \\ &= -\delta_{ij} + \frac{\gamma_{ij}}{w_i} + w_j. \end{aligned} \tag{5.2.5}$$

The compensated price elasticities are thus obtained by simply dividing s_{ij} by the average budget share $\bar{w}_i$ and, consequently, the asymptotic variances of the (compensated) price elasticities will be equal to those of the Slutsky coefficients divided by the squared average budget share:

$$V\left(\eta_{ij}\right) = V\left(\frac{s_{ij}}{\bar{w}_i}\right) = \frac{1}{\bar{w}_i^2} V\left(s_{ij}\right).$$

The expenditure (or income) elasticities have been defined as:

$$\varepsilon_i = 1 + \frac{\beta_i}{w_i}$$

and can be computed directly from the β_i estimates. Their asymptotic variances are again the same as those of the coefficient estimates divided by the squared average budget share:

$$V\left(\varepsilon_i\right) = V\left(1 + \frac{\beta_i}{\bar{w}_i}\right) = \frac{1}{\bar{w}_i^2} V\left(\beta_i\right).$$

In Tables 5.9 to 5.11 compensated price and expenditure elasticities are presented. The plausibility problems we encountered with the Slutsky matrix estimates are also present in the elasticities estimates. Part of the observations done in the previous section are thus still valid, in particular that own-price elasticities are all negative and cross-price elasticities are positive (with the exception of the US equation in unconstrained estimation). Expenditure (or budget) elasticities are all positive with the exception, again, of the unconstrained US equation.

	Static model					Autoregressive-errors model					Partial-adjustment model				
	η_{i1}	η_{i2}	η_{i3}	η_{i4}	ε_i	η_{i1}	η_{i2}	η_{i3}	η_{i4}	ε_i	η_{i1}	η_{i2}	η_{i3}	η_{i4}	ε_i
Germany	-0.26	0.18	0.17	0.14	1.31	-0.26	0.18	0.18	0.14	1.31	-0.26	0.17	0.16	0.14	1.32
	(0.0170)	(0.0060)	(0.0130)	(0.0050)	(0.0270)	(0.0170)	(0.0060)	(0.0140)	(0.0050)	(0.0270)	(0.0290)	(0.0110)	(0.0220)	(0.0070)	(0.0460)
	-15.29	30.00	13.08	28.00	48.52	-15.29	30.00	12.86	28.00	48.52	-8.97	15.45	7.27	20.00	28.70
France	0.65	-0.60	0.12	0.13	1.20	0.65	-0.60	0.11	0.13	1.20	0.68	-0.59	0.14	0.13	1.25
	(0.0420)	(0.0140)	(0.0310)	(0.0120)	(0.0640)	(0.0420)	(0.0140)	(0.0320)	(0.0120)	(0.0650)	(0.0510)	(0.0200)	(0.0390)	(0.0130)	(0.0820)
	15.48	-42.86	3.87	10.83	18.75	15.48	-42.86	3.44	10.83	18.46	13.33	-29.50	3.59	10.00	15.24
Italy	0.69	0.14	-0.65	0.12	1.17	0.65	0.13	-0.67	0.12	1.13	0.79	0.17	-0.56	0.13	1.34
	(0.0370)	(0.0120)	(0.0270)	(0.0100)	(0.0560)	(0.0360)	(0.0120)	(0.0300)	(0.0100)	(0.0570)	(0.0690)	(0.0270)	(0.0530)	(0.0170)	(0.1120)
	18.65	11.67	-24.07	12.00	20.89	18.06	10.83	-22.33	12.00	19.82	11.45	6.30	-10.57	7.65	11.96
Usa	-0.62	-0.20	-0.10	-1.05	-1.05	-0.60	-0.20	-0.09	-1.06	-0.99	-0.82	-0.23	-0.21	-1.06	-1.42
	(0.0730)	(0.0240)	(0.0550)	(0.0210)	(0.1130)	(0.0710)	(0.0230)	(0.0590)	(0.0200)	(0.1130)	(0.1010)	(0.0390)	(0.0780)	(0.0250)	(0.1630)
	-8.49	-8.33	-1.82	-50.00	-9.29	-8.45	-8.70	-1.53	-53.00	-8.76	-8.12	-5.90	-2.69	-42.40	-8.71

Table 5.9: Compensated price (η_{ij}) and budget (ε_i) elasticities, their standard-errors and t-statistics (unconstrained static, AR and PA models)

	Static model					Autoregressive-errors model					Partial-adjustment model				
	η_{i1}	η_{i2}	η_{i3}	η_{i4}	ε_i	η_{i1}	η_{i2}	η_{i3}	η_{i4}	ε_i	η_{i1}	η_{i2}	η_{i3}	η_{i4}	ε_i
Germany	-0.38	0.15	0.11	0.11	1.14	-0.37	0.15	0.11	0.11	1.13	-0.37	0.15	0.11	0.11	1.16
	(0.0085)	(0.0052)	(0.0111)	(0.0036)	(0.0145)	(0.0090)	(0.0049)	(0.0117)	(0.0034)	(0.0151)	(0.0125)	(0.0098)	(0.0167)	(0.0062)	(0.0219)
	-44.71	28.85	9.91	30.56	78.62	-41.11	30.61	9.40	32.35	74.83	-29.60	15.31	6.59	17.74	52.97
France	0.50	-0.63	0.04	0.10	0.98	0.52	-0.64	0.02	0.09	0.98	0.50	-0.63	0.05	0.08	1.00
	(0.0185)	(0.0113)	(0.0244)	(0.0078)	(0.0317)	(0.0185)	(0.0110)	(0.0245)	(0.0078)	(0.0317)	(0.0228)	(0.0178)	(0.0304)	(0.0115)	(0.0399)
	27.03	-55.75	1.64	12.82	30.91	28.11	-58.18	0.82	11.54	30.91	21.93	-35.39	1.64	6.96	25.06
Italy	0.54	0.11	-0.73	0.08	0.95	0.55	0.10	-0.74	0.09	0.96	0.53	0.11	-0.69	0.06	0.97
	(0.0165)	(0.0101)	(0.0218)	(0.0069)	(0.0283)	(0.0163)	(0.0088)	(0.0218)	(0.0063)	(0.0280)	(0.0329)	(0.0257)	(0.0439)	(0.0163)	(0.0575)
	32.73	10.89	-33.49	11.59	33.57	33.74	11.36	-33.94	14.29	34.29	16.11	4.28	-15.72	3.68	16.87
Usa	0.36	0.02	0.43	-0.82	0.42	0.31	0.02	0.49	-0.82	0.43	0.33	0.03	0.37	-0.75	0.24
	(0.0475)	(0.0291)	(0.0625)	(0.0202)	(0.0813)	(0.0475)	(0.0249)	(0.0631)	(0.0167)	(0.0810)	(0.0791)	(0.0617)	(0.1053)	(0.0386)	(0.1382)
	7.58	0.69	6.88	-40.59	5.17	6.53	0.80	7.77	-49.10	5.31	4.17	0.49	3.51	-19.43	1.74

Table 5.10: Compensated price (η_{ij}) and budget (ε_i) elasticities, their standard-errors and t-statistics (homogeneous static, AR and PA models)

	Static model					Autoregressive-errors model					Partial-adjustment model				
	η_{i1}	η_{i2}	η_{i3}	η_{i4}	ε_i	η_{i1}	η_{i2}	η_{i3}	η_{i4}	ε_i	η_{i1}	η_{i2}	η_{i3}	η_{i4}	ε_i
Germany	-0.39	0.14	0.14	0.11	1.11	-0.40	0.14	0.15	0.11	1.09	-0.38	0.14	0.14	0.10	1.15
	(0.0057)	(0.0033)	(0.0044)	(0.0030)	(0.0115)	(0.0055)	(0.0032)	(0.0044)	(0.0029)	(0.0114)	(0.0099)	(0.0053)	(0.0088)	(0.0055)	(0.0194)
	-68.42	42.42	31.82	36.67	96.52	-72.73	43.75	34.09	37.93	95.61	-38.38	26.42	15.91	18.18	59.28
France	0.48	-0.65	0.09	0.08	0.93	0.48	-0.65	0.09	0.08	0.92	0.48	-0.64	0.09	0.07	0.97
	(0.0111)	(0.0098)	(0.0086)	(0.0066)	(0.0250)	(0.0107)	(0.0097)	(0.0078)	(0.0060)	(0.0237)	(0.0178)	(0.0142)	(0.0175)	(0.0115)	(0.0376)
	43.24	-66.33	10.47	12.12	37.20	44.86	-67.01	11.54	13.33	38.82	26.97	-45.07	5.14	6.09	25.80
Italy	0.49	0.09	-0.66	0.08	0.89	0.52	0.09	-0.70	0.08	0.93	0.48	0.09	-0.61	0.04	0.92
	(0.0156)	(0.0090)	(0.0204)	(0.0075)	(0.0284)	(0.0155)	(0.0081)	(0.0207)	(0.0063)	(0.0274)	(0.0310)	(0.0181)	(0.0409)	(0.0200)	(0.0619)
	31.41	10.00	-32.35	10.67	31.34	33.55	11.11	-33.82	12.70	33.94	15.48	4.97	-14.91	2.00	14.86
Usa	0.53	0.12	0.11	-0.76	0.70	0.53	0.12	0.12	-0.76	0.78	0.51	0.10	0.05	-0.68	0.42
	(0.0151)	(0.0097)	(0.0102)	(0.0184)	(0.0659)	(0.0142)	(0.0091)	(0.0085)	(0.0158)	(0.0585)	(0.0275)	(0.0166)	(0.0281)	(0.0430)	(0.1428)
	35.10	12.37	10.78	-41.30	10.62	37.32	13.19	14.12	-48.10	13.33	18.55	6.02	1.78	-15.81	2.94

Table 5.11: Compensated price (η_{ij}) and budget (ε_i) elasticities, their standard-errors and t-statistics (homogeneous and symmetric static, AR and PA models)

The results across the models are similar, for all dynamic specifications and economic restrictions. In general, we observe that budget shares are inelastic with respect to prices, and elastic with respect to real expenditure. German (own- and cross-) price elasticities are the lowest, its own-price elasticities are situated between -0.26 (unconstrained estimation) and -0.40 (symmetric estimation), almost half compared to French and Italian results (French own-price elasticities swing between -0.59 and -0.65, Italian between -0.56 and -0.70). Cross-price elasticities are asymmetric: Germany has low cross-price elasticities while the other countries have high cross-price elasticities with respect to the German price. This is due to the fact that imports from Germany represent more than half of total imports (see Table 4.1) and consequently they are less sensitive to changes in other countries prices. In fact, if we look, for example, at the cross-price elasticities between France and Italy (whose shares of total imports are similar) we notice that they are very close. A historical tradition in trade exchanges exists between Switzerland (particularly its economically-strongest German region) and Germany. Such a link would have the effect of reducing the competition between countries and would also explain why German budget shares react elastically only to expenditure changes. We may then conclude that the asymmetric distribution of imports across countries has an impact on the cross-price elasticities. On the other hand, Germany has the highest expenditure elasticities and, in the homogeneous (and symmetric) framework, these elasticities are the only ones greater than unity. With unconstrained estimation, France and Italy also seem to react elastically to changes in expenditure; very inplausible is the (significant) negative expenditure elasticity of the US equation. Then, since the variations of import shares are mainly dictated by variations in real expenditure, in own-prices and in the German relative price, (but not in other country prices), we tend to conclude that competition between countries does not really exist.

If we compare the results under the different economic assumptions, we notice, in general, that the introduction of homogeneity and symmetry has the effect on the one side, to increase the own-price elasticities and, on the other side, to reduce the cross-price and expenditure elasticities. Thus, restricting the model has the consequence of reducing the impact of changes in cross-prices or real expenditure, and augmenting the impact of own-price variations.

Unconstrained Estimation: Only imports from the US seem to respond elastically (with an elasticity greater than unity) to changes in own-prices. German elasticities are, on the other hand, the more inelastic. The dynamic specification seems not to have a big impact on estimates; the results are similar across the models. Own-price elasticities for Germany are situated around -0.26, for France between -0.59 and -0.60, for Italy between -0.56 and -0.67, and for the US between -1.05 and -1.06. As we mentioned before, the results of the US equation concerning either price or expenditure elasticities are strongly inplausible.

Homogeneous Estimation: Compared to the unconstrained estimates, we observe (in absolute value) an increase in own price-elasticities and a decrease in cross-price and expenditure elasticities. German own-price elasticities are situated between -0.37 and -0.38 and are still the lowest; cross-price elasticities are between 0.14 and 0.18. Compared to the German results, French and Italian own-price elasticities are (in absolute value) between 70% and 100% higher, and their cross-price elasticities, with respect to the German relative price, are more than three times their corresponding elements in the German equation. In contrast, cross-price elasticities with respect to French or Italian prices are negligible. Hence, French and Italian budget shares respond elastically only to changes in own-prices and German prices, in addition to the variations in the real expenditure term. Concerning the US equation, the elasticities are only partly significant, but the introduction of homogeneity corrects the negative sign of the cross-price elasticities and thus produces more plausible results.

Homogeneous and Symmetric Estimation: The results confirm our previous suspicions: budget shares are, in general, inelastic with respect to (cross-) prices and elastic with respect to real expenditure. The smallest price elasticities and the highest expenditure elasticities (the only one greater than unity) can be found in the German equation. We mentioned before that this is probably due to the "dominant position" of German imports and, therefore, their variations are mainly dictated by changes in expenditure rather than prices. French and Italian budget shares are much more sensitive to changes in own- and cross-prices. Their own-price elasticities are almost double and their cross-price elasticities are more than three times their German counterparts. We would think that a stronger competition exists between imports from these two countries, but if we look at their respective cross-price elasticities, we note that they are close to zero. Thus, French, Italian and US budget shares respond only to changes in own-prices, German relative price, and, of course, to changes in real expenditure.

5.2.4 Comparison of Results

In the previous section we provided a descriptive comparison of results. This is useful in understanding the consequences of the dynamic and economic restrictions, but it is not enough to measure these consequences and the differences that arise across the models. With some simple tools like the goodness-of-fit measures, information criteria and residual analysis, we will try to give a more precise idea about the performances of the different models.

We start our analysis by plotting, in Figures 5.1 to 5.3, the observed and fitted values computed from each of the models.[6] At first sight, the adjustment seems to be good, at least for the dynamic models. It is, however, difficult to evaluate the differences produced by the alternative dynamic specifications and the economic hypotheses. The graphs suggest that the more restricted the model is, the worse is the adjustment to observed data. We will see below if these first impressions will be confirmed by the goodness-of-fit measures.

Goodness-of-fit measures are more useful to compare the results obtained with the alternative dynamic specifications, but not under the different regularity restrictions. Constrained estimation has the effect of producing residuals that are greater than the unconstrained ones, and consequently, the individual R^2 will be smaller. As we mentioned at the beginning of the chapter, the number of estimated parameters is different for each of the models. It is, thus, interesting to also compute the degrees-of-freedom adjusted $\bar{R}^2$:

$$\bar{R}_i^2 = 1 - \frac{T-1}{T-k}(1 - R_i^2), \tag{5.2.6}$$

[6] The first row of graphs in Figures 5.1 to 5.3 corresponds to the static model, the second to the autoregressive-errors model and the third to the partial-adjustment model fitted values.

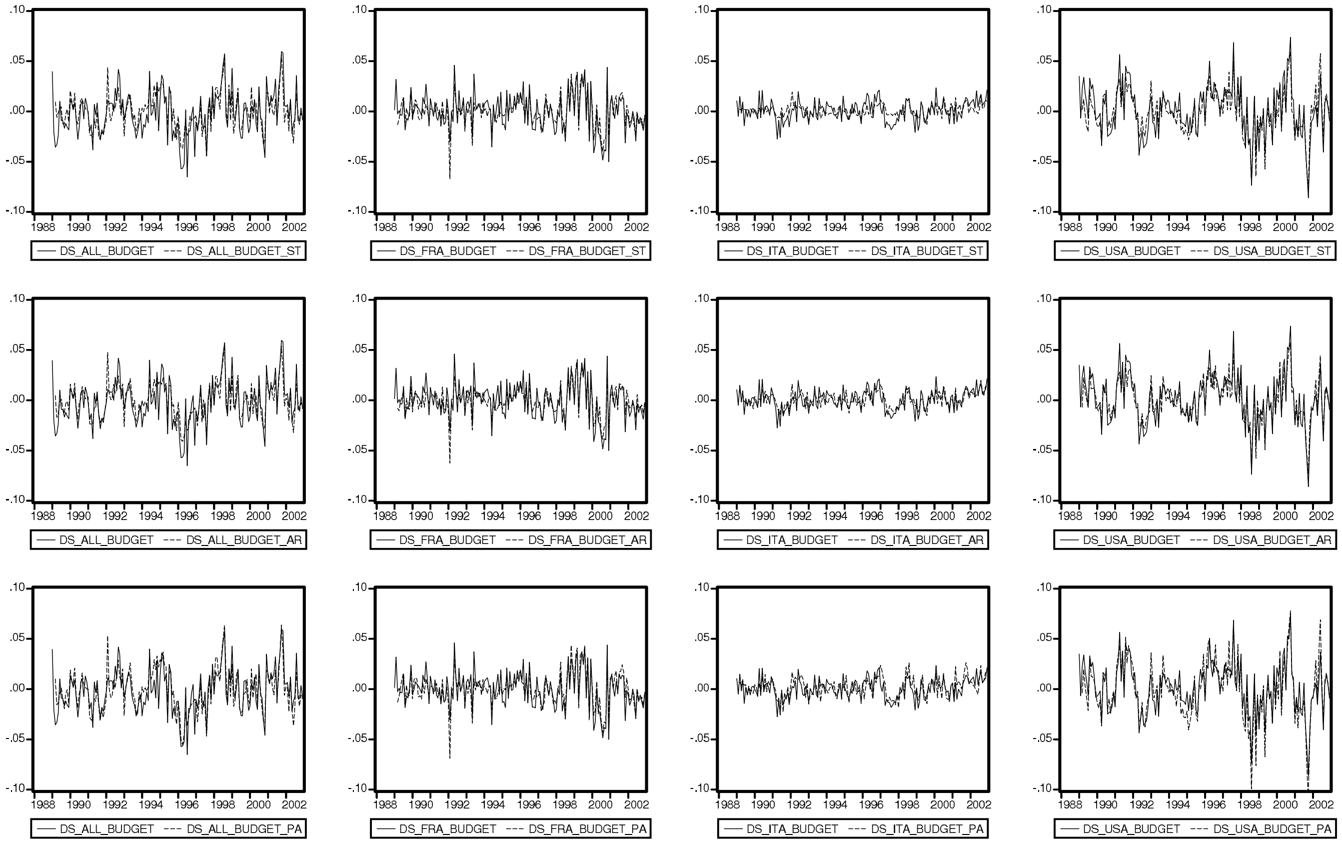

Figure 5.1: Unconstrained static, autoregressive-errors and partial-adjustment fitted and observed values, 1989-2002

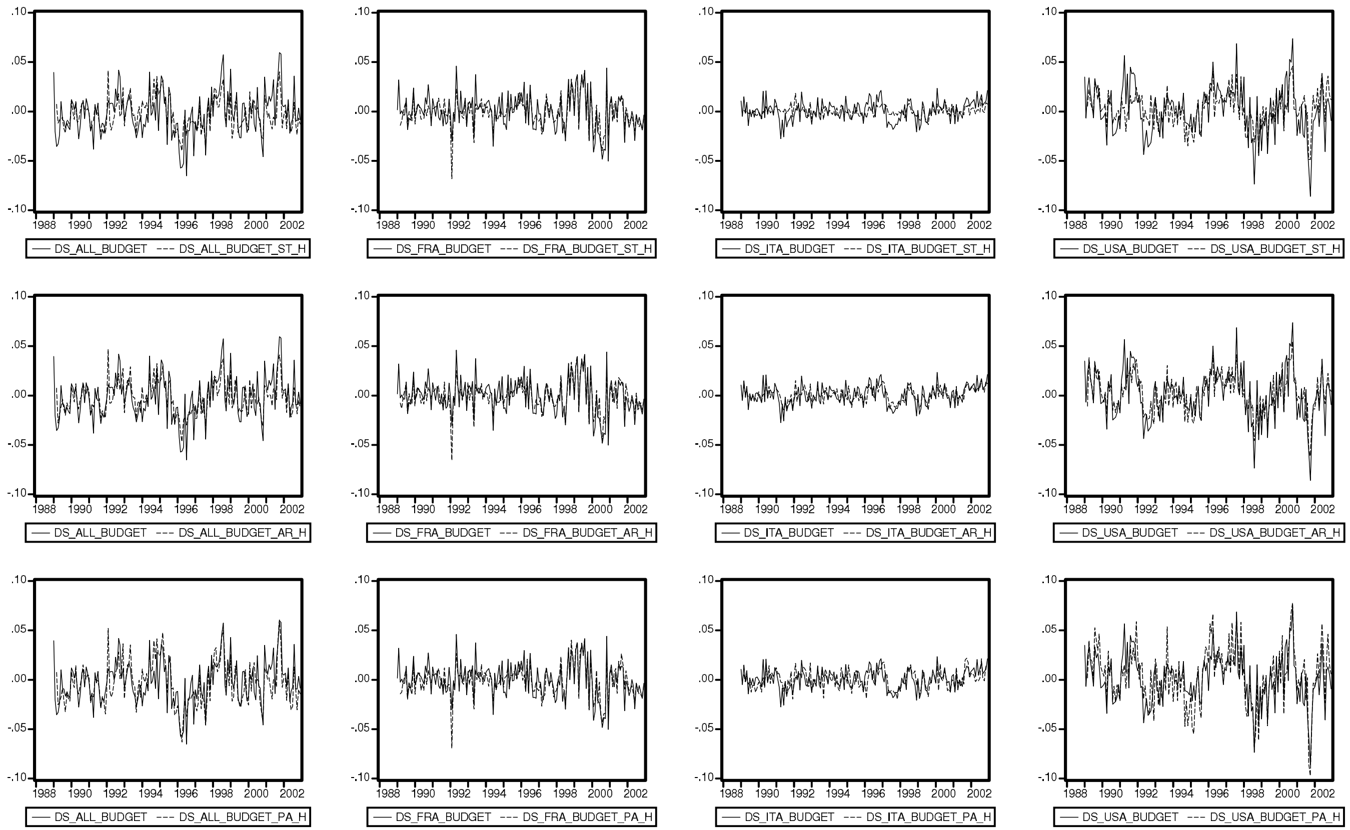

Figure 5.2: Homogeneous static, autoregressive-errors and partial-adjustment fitted and observed values, 1989-2002

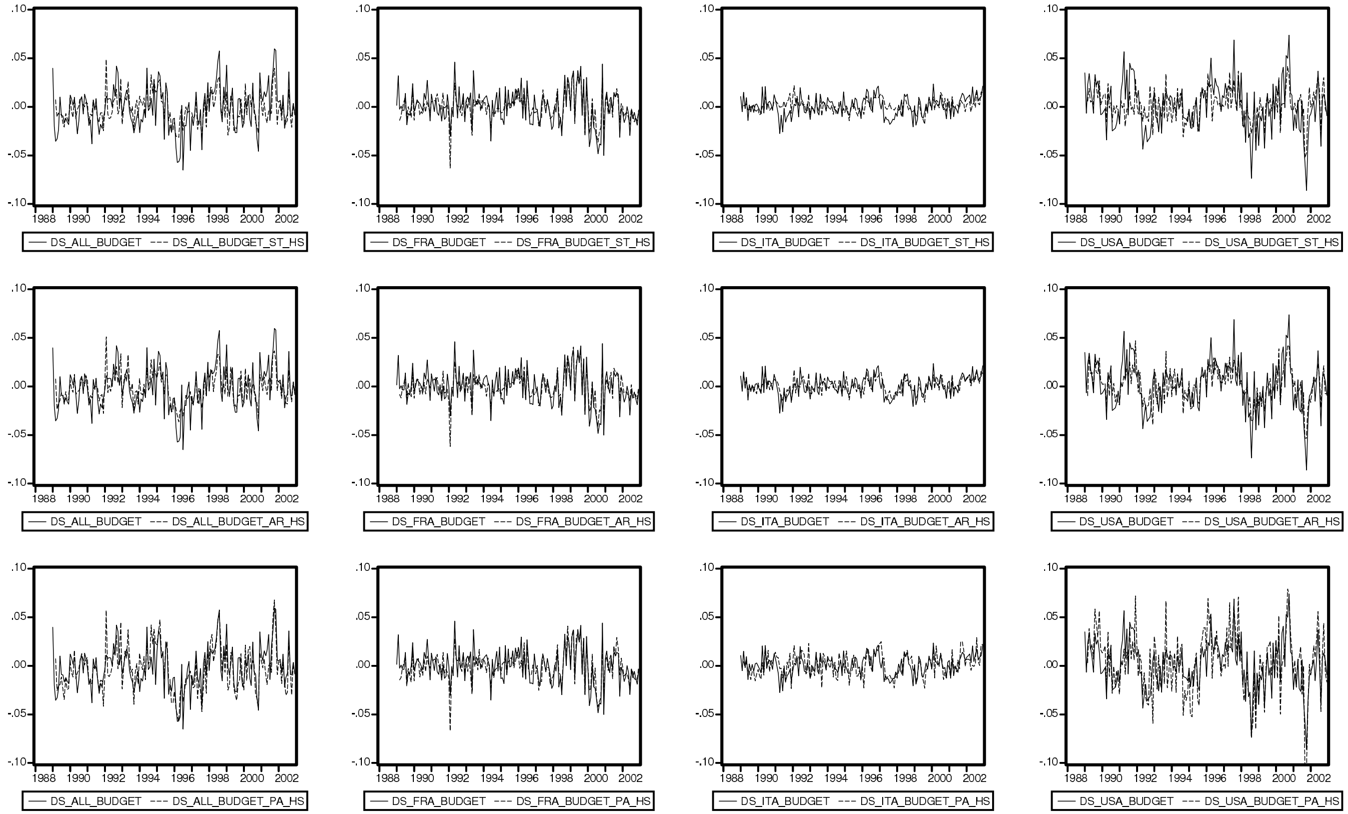

Figure 5.3: Symmetric static, autoregressive-errors and partial-adjustment fitted and observed values, 1989-2002

where T is the number of observations and k the number of estimated parameters. The results are summarized in Table 5.12.

Unconstrained Estimation: If we look at the unadjusted R^2, the best fit is provided by the partial-adjustment model, in particular for the German, French and Italian equations, but good results are also obtained for the last equation (USA). If we consider the adjusted $\bar{R}^2$, the best results are obtained with the partial-adjustment model for the German and Italian equations, with the static model for the French equation and with the autoregressive-errors model for the US equation. However, the fit is, in general, similar with the different dynamic specifications. The exception is the Italian equation, for which we observe sensible differences across the models. Italy is also the country for which we have the worst adjustment. For the other equations, the fit is good, taking into consideration the number of observations (166) compared to the number of estimated parameters (15 for the static model, 33 for the autoregressive-errors and partial-adjustment models). The result of the static model is somehow surprising; as a simple specification fits, at least, as well as the other models.

Homogeneous Estimation: The autoregressive-errors model presents the highest R^2 and $\bar{R}^2$, followed by the static model which provides the best $\bar{R}^2$ for the French equation. Again, unlike its simplicity, the static model fit is good. The consequences of the homogeneity restrictions are very different across the equations. As expected, the homogeneous R^2 are smaller compared to unconstrained estimation. This can be observed particulary in the Italian and US equations, for which the R^2 (and $\bar{R}^2$) have fallen by 80% (except for the Italian equation in the autoregressive-errors model). The decrease in the R^2 for the German

Unconstrained estimation

	R^2			Adjusted $\bar{R}^2$		
	ST	**AR**	**PA**	**ST**	**AR**	**PA**
Germany	0.62	0.62	0.67	0.58	0.53	0.59
France	0.71	0.71	0.72	0.68	0.64	0.65
Italy	0.32	0.31	0.48	0.26	0.14	0.35
USA	0.78	0.83	0.75	0.76	0.79	0.68

Homogeneous estimation

	R^2			Adjusted $\bar{R}^2$		
	ST	**AR**	**PA**	**ST**	**AR**	**PA**
Germany	0.51	0.58	0.51	0.47	0.49	0.40
France	0.68	0.71	0.68	0.66	0.65	0.61
Italy	0.23	0.45	0.23	0.18	0.33	0.07
USA	0.50	0.64	0.42	0.46	0.56	0.30

Homogeneous and symmetric estimation

	R^2			Adjusted $\bar{R}^2$		
	ST	**AR**	**PA**	**ST**	**AR**	**PA**
Germany	0.48	0.54	0.47	0.46	0.46	0.37
France	0.67	0.70	0.66	0.65	0.65	0.60
Italy	0.17	0.44	0.01	0.13	0.33	-0.17
USA	0.41	0.56	0.22	0.38	0.48	0.08

Table 5.12: Single equation R^2 and adjusted $\bar{R}^2$

equation is moderate and practically negligible for the French one. It is difficult to explain the origin of these wide differences. It could be that the homogeneity restrictions are appropriate only for some equations.

Homogeneous and Symmetric Estimation: The results indicate that the equation fit is, for the German, French and US equations, only slightly affected by the symmetry restrictions: the adjusted $\bar{R}^2$ is situated between 0.37 and 0.46 for Germany, between 0.60 and 0.65 for France and between 0.38 and 0.48 (if we do not consider the poor result of the partial-adjustment model) for the USA. The impact is larger for the Italian equation, in particular with the partial-adjustment model where the R^2 is close to zero (and the adjusted $\bar{R}^2$ is negative). Otherwise, the remarks made for the homogeneous estimates are still valid, with the best results (for both the unadjusted and adjusted R^2) provided by the autoregressive-errors model.

The usual R^2 measure is, however, truly appropriate only in a single equation context. Several alternatives for system estimation have been proposed (see for example Judge et al., 1985), between these we chose the generalizations of the R^2 proposed by McElroy and Berndt. McElroy's R^2_m, with maximum likelihood estimation (see Edgerton et al, 1996), can be expressed as:

$$R^2_m = 1 - \frac{n-1}{\mathrm{tr}\ \hat{\Omega}^{-1}\hat{\Omega}_w}, \tag{5.2.7}$$

where $n-1$ denotes the number of estimated equations, $\hat{\Omega}$ the estimated residual covariance matrix and $\hat{\Omega}_w$ the estimated covariance matrix of the dependent variables. Berndt's R^2_b measure replaces the trace function by the determinant and can be written as:

$$R_b^2 = 1 - (n-1)\sqrt{\frac{\det \hat{\Omega}}{\det \hat{\Omega}_w}}. \tag{5.2.8}$$

The advantage of these measures is that they reduce to the usual R^2 statistic in the single equation case. Nevertheless, they should be taken as a complement to the single equation R^2 and can not replace it. Degrees-of-freedom adjusted coefficients can be computed also for systemwide R^2 in a similar manner to the single equation case (see equation (5.2.6)).

Unconstrained Estimation: The results presented in Table 5.13 confirm our previous suspicions, with the partial-adjustment model having the highest R^2 and $\bar{R}^2$. The worst fit is given by the autoregressive-errors model, but, in general, the measures are very close to each other.

Homogeneous Estimation: We have exactly the opposite situation of unconstrained estimation: the highest adjusted (and unadjusted) McElroy and Berndt's R^2 are obtained with the autoregressive-errors model, the lowest with the partial-adjustment model. The comparison with the unconstrained results reveals that the fit has fallen to 15% for the McElroy's measure, and to more than 50% for the Berndt's measure. The particularly bad fit of the Italian and US equations penalizes the partial-adjustment model which shows the largest decreases.

Homogeneous and Symmetric Estimation: The systemwide goodness-of-fit measures are, except for the partial-adjustment model, practically unaffected by the symmetry constraints. If we compare the results of the symmetric and the homogeneous estimation, we notice that the differences are negligible for

Unconstrained estimation

	McElroy		Berndt	
	R_m^2	Adjusted $\bar{R}_m^2$	R_b^2	Adjusted $\bar{R}_b^2$
Static	0.79	0.77	0.54	0.50
Autoregressive-errors	0.79	0.73	0.54	0.43
Partial-adjustment	0.82	0.77	0.64	0.56

Homogeneous estimation

	McElroy		Berndt	
	R_m^2	Adjusted $\bar{R}_m^2$	R_b^2	Adjusted $\bar{R}_b^2$
Static	0.70	0.68	0.30	0.25
Autoregressive-errors	0.75	0.70	0.50	0.39
Partial-adjustment	0.70	0.64	0.29	0.14

Homogeneous and symmetric estimation

	McElroy		Berndt	
	R_m^2	Adjusted $\bar{R}_m^2$	R_b^2	Adjusted $\bar{R}_b^2$
Static	0.69	0.67	0.24	0.20
Autoregressive-errors	0.73	0.68	0.44	0.34
Partial-adjustment	0.68	0.62	0.16	0.002

Table 5.13: Systemwide McElroy and Berndt's R^2

the static and autoregressive-errors models. Concerning the partial-adjustment model, McElroy's R^2 follows the same trend of the other models, while Berndt's R^2 falls steeply.

An alternative to measure goodness-of-fit, besides the R^2, are the information criteria. Between these, we choose Akaike and Schwarz criteria defined as (Amemiya, 1985):

$$\begin{aligned} AIC &= -\frac{2\log L}{T} + \frac{2k}{T} \\ SC &= -\frac{2\log L}{T} + \frac{k\log T}{T} \end{aligned}$$

where $\log L$ denotes the maximized loglikelihood value, k, the number of parameters and T, the number of observations. Like the adjusted $\bar{R}^2$, information criteria are a way of trading off goodness-of-fit and parsimony of the models. According to their definition, we would select the model that minimizes these criteria. In Table 5.14 the values of the Akaike and Schwarz criteria are presented for each of the models.

Firstly, if we look at the dynamic specification, we notice that with unconstrained and homogeneous estimation, the autoregressive-errors model has the lowest Akaike and Schwarz criteria, while in homogeneous and symmetric estimation the best results are provided by the partial-adjustment model. Hence, the results partly contradict the conclusion we draw comparing the single equation and systemwide R^2. However, we should not attach too much importance to these results. In fact, if we look in more detail, we see that the values of the AIC and SC criteria are often very close. In the unconstrained estimation framework, the differences in the AIC and SC criteria between the three dynamic specifications do not exceed 1.8%. A similar situation is repeated with homogeneous (and symmetric) estimation between the autoregressive-errors and partial-adjustment models for the AIC criterion, and between the three

	k	**Loglikelihood**	**AIC**	**SC**
Static				
Unconstrained	15	1614.73	-19.274	-18.993
Homogeneous	12	1544.35	-18.462	-18.237
Symmetric	9	1529.24	-18.316	-18.147
Autoregressive-errors				
Unconstrained	33	1662.92	-19.638	-19.019
Homogeneous	30	1597.62	-18.887	-18.325
Symmetric	27	1581.43	-18.728	-18.222
Partial-adjustment				
Unconstrained	33	1654.55	-19.537	-18.918
Homogeneous	30	1590.64	-18.803	-18.241
Symmetric	27	1587.20	-18.798	-18.291

Table 5.14: Akaike (AIC) and Schwarz (SC) information criteria

models for the SC criterion. The introduction of the economic restrictions has similar consequences. When we impose homogeneity and/or symmetry, the effect is the same for all dynamic specifications: the AIC and SC criteria increase (or, in absolute value, decrease).

We will conclude this section with an analysis of residuals. In Figures 5.4 to 5.6, the residuals for each equation, under the different dynamic specifications and regularity assumptions, are represented. In general, the residuals pattern differs a lot – not only across the models but even across the equations. We see, for example, that the Italian residuals have limited variations, while the US residuals have high frequency fluctuations. We also notice that the economic restrictions have the consequence of augmenting the variability and the importance of the residuals, and also, in some cases, to reinforce a cyclical pattern which could be a signal of mis-specification. This is particularly true for the static (and it is not a surprise) and the partial-adjustment

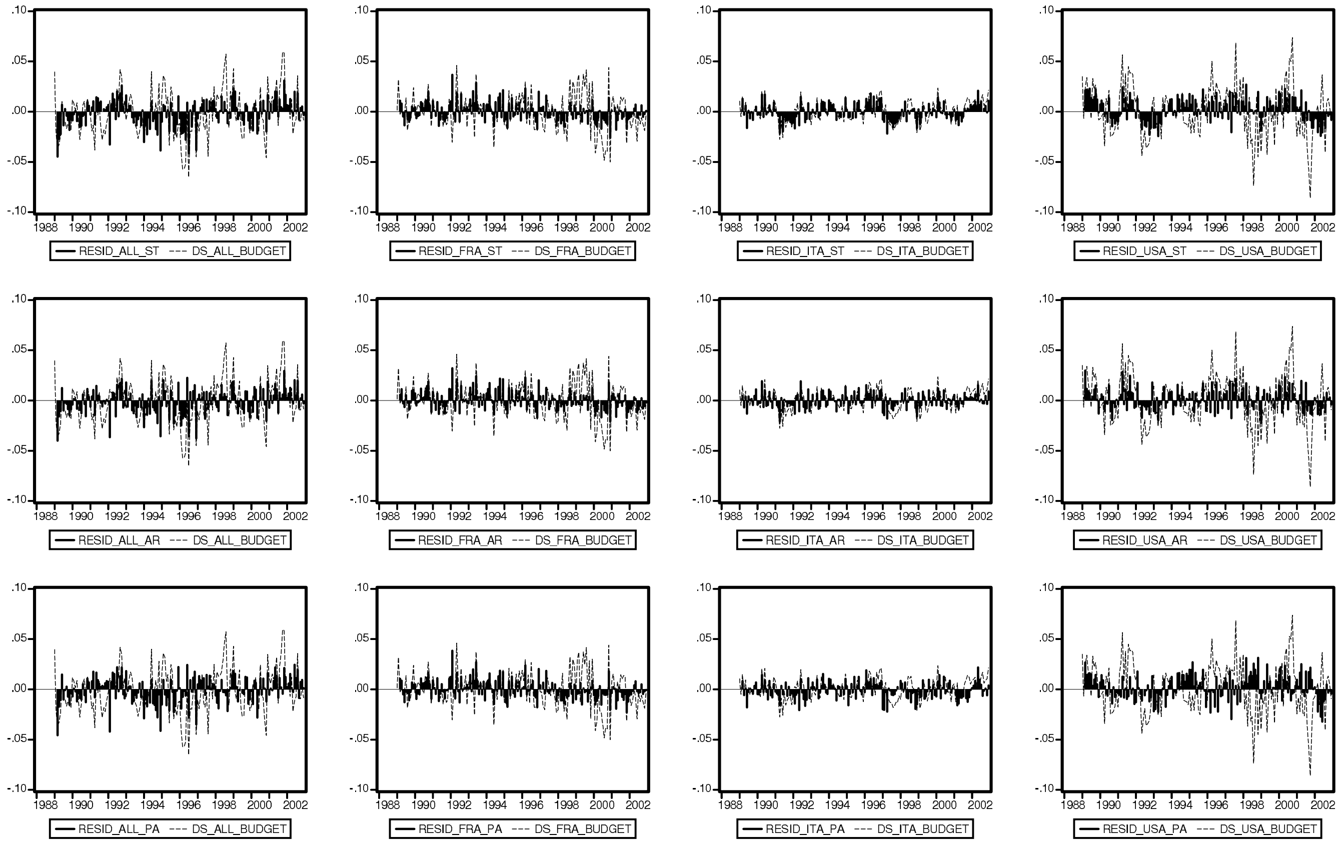

Figure 5.4: Plot of residuals: unconstrained static (ST), autoregressive-errors (AR) and partial-adjustment (PA) models, 1989-2002

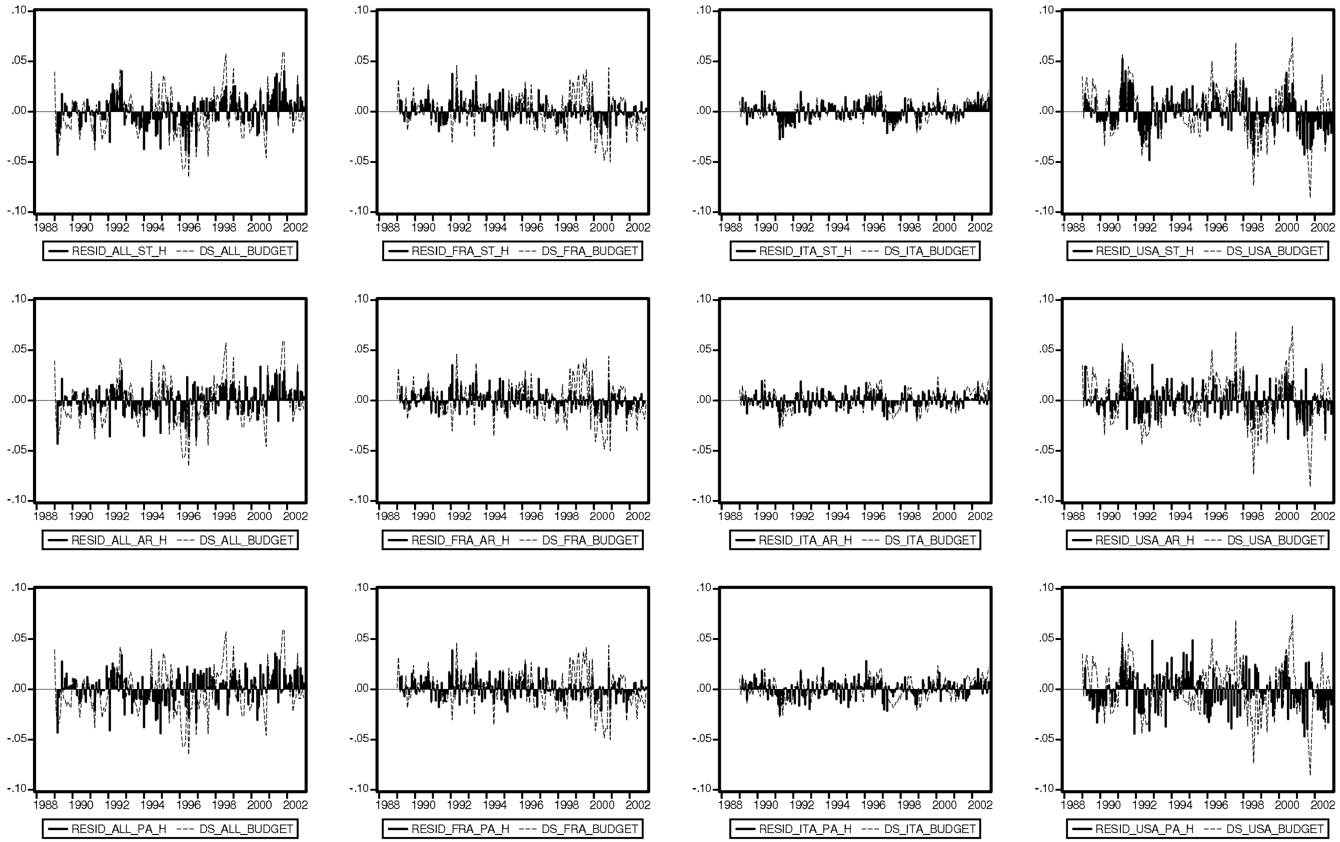

Figure 5.5: Plot of residuals: homogeneous static (ST), autoregressive-errors (AR) and partial-adjustment (PA) models, 1989-2002

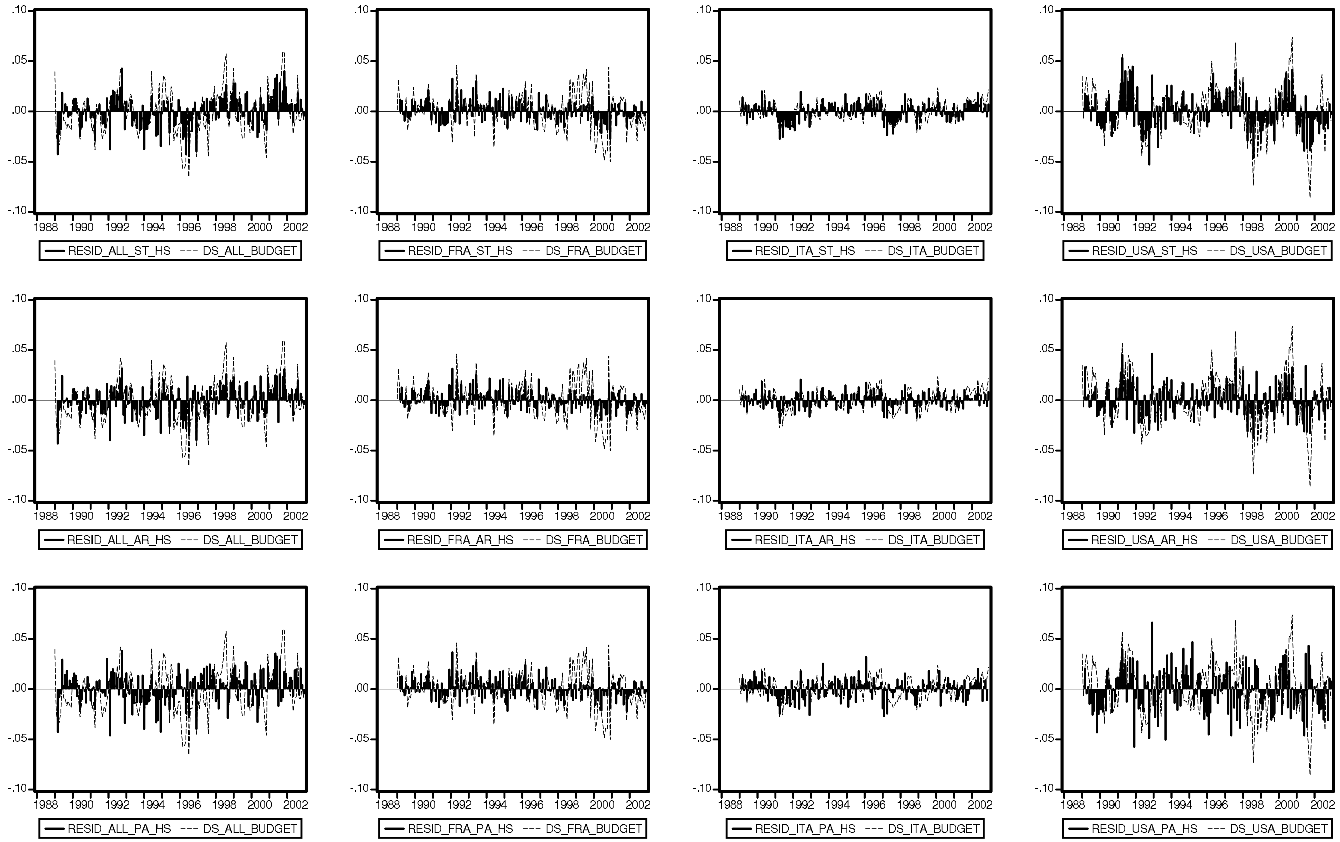

Figure 5.6: Plot of residuals: symmetric static (ST), autoregressive-errors (AR) and partial-adjustment (PA) models, 1989-2002

models. The autoregressive-errors model seems to have residuals whose pattern is better behaved (except perhaps for the US equation). Looking at the graphical representation we can not, then, exclude mis-specification problems which could result in residuals serial correlation.

In Section 4.5, we used diagnostic tests to determine the polynomial lag length for the endogenous and exogenous vectors of variables. With $p = 2$ and $q = 0$, the OLS residuals of the general dynamic model were normal, not serially correlated nor heteroskedastic. To check whether or not the restrictions imposed on the general dynamic model have an influence on the residuals properties, we should compute the diagnostic tests again. This will not be done here because we don't know exactly what the impact of the different dynamic and economic restrictions on the diagnostic test distribution is (for example the Breusch-Godfrey test statistic is not valid with the cross-equation restrictions introduced by symmetry). We will, however, test the presence of autocorrelation with an alternative and valid procedure based on the regression of the residuals on their lagged values:

$$\hat{u}_{it} = a + \rho_1 \hat{u}_{i,t-1} + \ldots + \rho_s \hat{u}_{i,t-s} + \varepsilon_{it} \tag{5.2.9}$$

and check, with an F-test, the joint significance of the ρ_i ($i = 1, \ldots, s$) coefficients (i.e. we test the null hypothesis $H_0 : \rho_1 = \ldots = \rho_s = 0$ against a bilateral alternative).[7] In Tables 5.15 to 5.17, the p-values of the F-test statistics on the (individual) residuals autoregressive processes (of order 1 to 6) are summarized.

Concerning heteroskedasticity and normality, the first seems to be excluded (there is no evidence of heteroskedasticity in the graphical representations), while the second is valid only asymptotically.

[7] Under the null hypothesis, the residuals are not serially correlated.

	Lag					
	$s=1$	$s=2$	$s=3$	$s=4$	$s=5$	$s=6$
Static						
Germany	0.0026	0.0003	0.0013	0.0009	0.0008	0.0001
France	0.5340	0.0835	0.1193	0.0862	0.1516	0.2004
Italy	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
USA	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Autoregressive-errors						
Germany	0.9835	0.8882	0.9121	0.9106	0.4044	0.1294
France	0.5741	0.8254	0.8832	0.6983	0.8268	0.8087
Italy	0.5196	0.7298	0.4001	0.4969	0.6043	0.7290
USA	0.4916	0.6479	0.7609	0.6615	0.0335	0.0194
Partial-adjustment						
Germany	0.1356	0.3586	0.2398	0.1357	0.0634	0.0077
France	0.1694	0.1651	0.2204	0.1645	0.2640	0.3168
Italy	0.0601	0.1191	0.0053	0.0056	0.0092	0.0155
USA	0.0009	0.0000	0.0000	0.0000	0.0000	0.0000

Table 5.15: p-values of the F-test on the individual residuals autoregression processes of order s (unconstrained estimation)

Unconstrained Estimation: The results seem to confirm our suspicions. There is evidence of autocorrelation in the static and partial-adjustment models: the smooth cyclical pattern which appears clearly in the German and US equation residuals is confirmed by the results of Table 5.15. The null hypothesis can always be rejected (at both 5% and 1%) for the German, Italian and US equations of the static model, and for the US and (partly) Italian equations of the partial-adjustment model. In the autoregressive-errors residuals we do not recognize any regular or cyclical pattern and the null hypothesis of the serial correlation test is never rejected.

Homogeneous Estimation: The introduction of the homogeneity restrictions produces fluctuations of higher amplitude. The implications are different between

	Lag					
	$s = 1$	$s = 2$	$s = 3$	$s = 4$	$s = 5$	$s = 6$
Static						
Germany	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
France	0.8715	0.0306	0.0640	0.0734	0.1309	0.2194
Italy	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
USA	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Autoregressive-errors						
Germany	0.9083	0.9409	0.8871	0.8056	0.3003	0.0961
France	0.5738	0.8237	0.8638	0.8349	0.8964	0.9093
Italy	0.5839	0.8616	0.6866	0.8067	0.7728	0.8149
USA	0.7054	0.9041	0.8921	0.8419	0.4535	0.5588
Partial-adjustment						
Germany	0.0426	0.0841	0.0312	0.0199	0.0205	0.0058
France	0.6960	0.1425	0.2660	0.2678	0.3991	0.5256
Italy	0.2610	0.2772	0.0725	0.0955	0.0135	0.0305
USA	0.1186	0.0004	0.0006	0.0017	0.0024	0.0053

Table 5.16: p-values of the F-test on the individual residuals autoregression processes of order s (homogeneous estimation)

the individual equations. We discussed before that restricting the model increases the sum of squared residuals and this fact can be observed graphically. The autoregressive-errors model is also affected, in particular for the US equation residuals. The null hypothesis of the F-test is, however, not rejected and the presence of serial correlation can be excluded. It is not excluded (the null hypothesis is rejected at 1%) for the German, Italian and US equations of the static model and partly for the US equation of the partial-adjustment model. Homogeneity, thus, solved part of the residuals autocorrelation problem of the Italian equation in the partial-adjustment model. On the other hand, for the German equation, we can now not reject, at 5%, the presence of autocorrelation.

	Lag					
	$s=1$	$s=2$	$s=3$	$s=4$	$s=5$	$s=6$
Static						
Germany	0.0000	0.0000	0.0001	0.0001	0.0001	0.0000
France	0.9660	0.0189	0.0423	0.0444	0.0670	0.1164
Italy	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
USA	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Autoregressive-errors						
Germany	0.9476	0.9332	0.9245	0.8882	0.2949	0.1296
France	0.4374	0.7382	0.8791	0.7613	0.8535	0.8695
Italy	0.5831	0.7852	0.8266	0.8748	0.8340	0.8614
USA	0.7919	0.9474	0.7922	0.8688	0.0505	0.0927
Partial-adjustment						
Germany	0.1421	0.2822	0.1488	0.1065	0.1683	0.0651
France	0.8497	0.1535	0.2775	0.2741	0.3410	0.4620
Italy	0.2104	0.3207	0.1061	0.1080	0.0143	0.0325
USA	0.6226	0.0608	0.1009	0.1687	0.1828	0.2702

Table 5.17: p-values of the F-test on the individual residuals autoregression processes of order s (homogeneous and symmetric estimation)

Symmetric and Homogeneous Estimation: The impact of symmetry is negligible for the static and autoregressive-errors models. For the partial-adjustment model, the major improvement is given by the elimination of the serial correlation problem: at 1%, there is not any evidence of autocorrelation.

We will conclude this section by summarizing the main results we obtained:

1. **Goodness-of-Fit:** The fit of the different dynamic specifications is, in general, similar. The partial-adjustment model for the R^2 and, partly, the autoregressive-errors model for the adjusted $\bar{R}^2$ show the best results with unconstrained estimation. Homogeneity reduces the adjustment quality and has, in particular, serious consequences on the fit of the partial-adjustment model. The

autoregressive-errors model has the best (adjusted) results both under homogeneity and symmetry. However, symmetry generally has a negligible impact on the goodness-of-fit measures.

2. **Information Criteria:** The results are very close for the static and partial-adjustment models. Strictly following these criteria, we would choose the autoregressive-errors model with unconstrained and homogeneous estimation and the partial-adjustment model under homogeneity and symmetry. Imposing the economic restrictions has a negative impact (in absolute value) on the criteria.

3. **Residual Analysis:** In general, the residuals seem to be well-behaved for the autoregressive-errors and partial-adjustment models. We can exclude the presence of heteroskedasticity but not serial correlation: the cyclical pattern of some series may be interpreted as a mis-specification problem that can result in residuals autocorrelation. The autocorrelation tests we computed suggest the presence of serial correlation in the static and, partly, partial-adjustment residuals.

5.3 Hypothesis Testing

In the previous section we investigated the consequences for estimation of, on the one hand, the economic restrictions (homogeneity and symmetry) and, on the other hand, the restrictions imposed to the general dynamic specification (which reduces to the partial-adjustment model since $q = 0$) to obtain the static and autoregressive-errors models. The aim of this section is to test whether or not the imposed restrictions are significant. Firstly, we will test homogeneity and symmetry and secondly, test the restrictions imposed on the dynamic specification.

All of these tests will be computed using the likelihood ratio criterion presented in Section 3.3. We wrote the loglikelihood test statistic as:

$$LR = -2\left(L_{H_0} - L_{H_1}\right)$$

where L_{H_0} and L_{H_1} are, respectively, the maximized loglikelihood under the null and under the alternative (or unconstrained) hypothesis. In Chapter 3, the loglikelihood function (for the unrestricted error-correction model) has been defined as:

$$L(\Pi, \Psi, \Phi_1, \ldots, \Phi_p, \Omega) = -\frac{(n-1)T}{2}\log 2\pi - \frac{T}{2}\log|\Omega| - \frac{1}{2}\sum_{t=1}^{T} u_t'\Omega^{-1}u_t$$

with $u_t = \Phi(L)w_t - \Phi(1)\left[\Pi x_{t-1} + \Psi \triangle x_t\right]$ and $\Omega = 1/T\sum_{t=1}^{T} u_t u_t'$. The constrained loglikelihoods can be easily obtained from this one.

Asymptotically, the LR test statistic follows, under the null hypothesis, a χ_r^2 distribution, where r is the number of restrictions. It is, however, well known that the LR test statistic is biased towards rejection of the null hypothesis in small sample situations (see for example Laitinen, 1978). For this reason, we will also compute a small-sample correction based on the Anderson (for the uniform mixed linear constraints tests, i.e. homogeneity and the dynamic specification tests) and the Italianer (for the nonlinear restrictions tests, i.e. symmetry) correction factors. The Anderson (1958) small sample correction factor is defined by:

$$\tau_1 = \frac{T - q_2 - \frac{1}{2}(g + q_1 + 1)}{T},$$

where k is the number of columns of the regressor matrix, q_1 the number of constrained columns, $q_2 = k - q_1$, and g is the number of rows of w_t. The corrected LR

statistic $-2\tau_1 \log\lambda$ follows a $\chi^2_{gq_1}$ distribution. Italianer's (1985) correction factor can be written as:

$$\tau_2 = \frac{1}{2}\left(f_U + f_R\right),$$

where $f_U = \left(T - k - \frac{1}{2}(g+1)\right)/T$ and $f_R = \left(T - q_2 - \frac{1}{2}(g+1)\right)/T$. The corrected LR test statistic follows a χ^2_r, where r is the number of imposed restrictions.

5.3.1 Homogeneity

We imposed homogeneity by introducing relative prices and setting the last column of the price coefficients matrix (the one corresponding to the reference price) equal to the null vector. If we rewrite the regressors matrix (with relative prices) by putting the reference price in the first position,

$$\triangle_s x_t^h = \begin{pmatrix} \triangle_s \ln p_t^{usa} \\ \triangle_s \ln \frac{p_t^{all}}{p_t^{usa}} \\ \triangle_s \ln \frac{p_t^{fra}}{p_t^{usa}} \\ \triangle_s \ln \frac{p_t^{ita}}{p_t^{usa}} \\ \triangle_s \ln \frac{M_t}{P_t} \end{pmatrix},$$

the equilibrium multipliers matrix can be written as $\Pi^h = \begin{pmatrix}\Pi^{**} & \Pi^{*}\end{pmatrix}$, with:

$$\Pi^{**} = \begin{pmatrix} \sum_{j=1}^n \gamma_{1j} \\ \sum_{j=1}^n \gamma_{2j} \\ \sum_{j=1}^n \gamma_{3j} \end{pmatrix} \qquad \text{and} \qquad \Pi^{*} = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \beta_1 \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \beta_2 \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \beta_3 \end{pmatrix}.$$

Testing homogeneity, thus, reduces to test the null hypothesis $H_0 : \Pi^{**} = 0$ against the alternative $H_1 : \Pi^{**} \neq 0$.

In Table 5.18, the constrained (under homogeneity) and unconstrained loglikelihood values are summarized, as are the loglikelihood test statistics (LR and adjusted

LR, using the Anderson small-sample correction) for each of the dynamic specifications.

	L_{H_0}	L_{H_1}	LR stat	Adj LR stat	$\chi^2_{3,0.95}$	$\chi^2_{3,0.99}$
Static	1544.35	1614.73	140.75	135.24		
Autoregressive-errors	1597.62	1662.92	130.61	120.77	7.82	11.34
Partial-adjustment	1590.64	1654.55	127.82	118.20		

Table 5.18: Likelihood ratio (LR) statistics for the homogeneity test

As we can see, homogeneity is strongly rejected for all the models. The amplitude with which the hypothesis is rejected is surprising. In the literature, the rejection of the homogeneity restriction has often been related to the dynamic mis-specification. Here, we notice that the introduction of dynamics has practically no influence on the test results. The test statistics for the autoregressive-errors and partial-adjustment models are lower compared to the static model, but homogeneity is still clearly rejected. Thus, if on the one hand homogeneity increases the plausibility of our estimations (as we showed in the previous section), then on the other hand it is rejected by data.

Homogeneity can also be tested in two alternative ways. The first way consists of estimating the system including the omitted variable ($\triangle_s \ln p_t^{usa}$) and testing its coefficient significance in the individual equations. The second way applies only to the static model and it is the F-statistic presented by Laitinen (1978).

Table 5.19 summarizes the coefficient estimates ($\sum_{j=1}^{4} \gamma_{ij}$) of the added variable ($\triangle_s \ln p_t^{usa}$) and the corresponding Slutsky coefficients (s_{i4}) and price elasticities (η_{i4}). As we can see, the coefficients (equilibrium multipliers, Slutsky coefficients and price elasticities) are all significant, thus contradicting the homogeneity assumptions. Therefore, the results of Table 5.19 reinforce the conclusions drawn with the LR test statistic: homogeneity is rejected by data.

The explanatory power of the omitted variable is confirmed by the R^2 summarized in Table 5.20. If we compare the adjusted R^2 of the unconstrained relative prices model and the homogeneous model, we notice that it is larger for the unconstrained model. The omitted variable should, then, be included in the system. This is particularly true for the Italian and US equations that, under homogeneity, have bad performances.

In the context of the static model, homogeneity can be tested using the F-statistic proposed by Laitinen (1978). Consider the unconstrained model (in absolute prices), and let π_i denote the i-line of the coefficient matrix Π. For equation i, homogeneity takes the form $\pi_i a = 0$, where $a = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \end{pmatrix}'$. For all equations, homogeneity requires that $\Pi a = 0_{3\times 1}$, or equivalently, by applying the vec operator to both sides,

$$R \text{ vec } \Pi = 0_{3\times 1} \tag{5.3.1}$$

with $R = a' \otimes I$. If (5.3.1) holds, Laitinen showed that the F-statistic,

$$F_{obs} = \frac{\text{vec}' \,\Pi\, R' \,\widehat{\Omega}^{-1}\, R \text{ vec } \Pi}{a'(X'X)^{-1}a} \tag{5.3.2}$$

is distributed as:

$$\frac{(n-1)(T-n-1)}{T-2n+1} F_{n-1,T-2n+1}. \tag{5.3.3}$$

Using our data, $F_{obs} = 221.57$, and the critical value, at a 5% significance level, is $3.04(F_{3,161,0.05}) = 7.8$. Homogeneity is, thus, clearly rejected.

The criteria and the test statistics we presented all give evidence for the rejection of homogeneity. As we mentioned before, these results are surprising because of

	Static model			**AR model**			**PA model**		
	$\sum_{j=1}^{4}\gamma_{ij}$	s_{i4}	η_{i4}	$\sum_{j=1}^{4}\gamma_{ij}$	s_{i4}	η_{i4}	$\sum_{j=1}^{4}\gamma_{ij}$	s_{i4}	η_{i4}
Germany	0.128	0.192	0.34	0.137	0.201	0.36	0.123	0.187	0.33
	(0.0176)	(0.0176)	(0.0314)	(0.0182)	(0.0182)	(0.0325)	(0.0303)	(0.0303)	(0.0541)
	7.27	10.91	10.82	7.53	11.04	11.08	4.06	6.17	6.10
France	0.049	0.068	0.41	0.048	0.067	0.40	0.060	0.079	0.48
	(0.0126)	(0.0126)	(0.0761)	(0.0128)	(0.0128)	(0.0773)	(0.0160)	(0.0160)	(0.0966)
	3.89	5.40	5.39	3.75	5.23	5.18	3.75	4.94	4.97
Italy	0.049	0.067	0.42	0.037	0.055	0.34	0.083	0.101	0.63
	(0.0107)	(0.0107)	(0.0669)	(0.0110)	(0.0110)	(0.0688)	(0.0209)	(0.0209)	(0.1308)
	4.58	6.26	6.27	3.36	5.00	4.94	3.97	4.83	4.82
Usa	-0.226	-0.327	-2.87	-0.222	-0.323	-2.84	-0.266	-0.367	-3.22
	(0.0151)	(0.0151)	(0.1326)	(0.0154)	(0.0154)	(0.1356)	(0.0218)	(0.0218)	(0.1911)
	-14.97	-21.66	-21.64	-14.38	-20.92	-20.94	-12.23	-16.87	-16.85

Table 5.19: Equilibrium multipliers, Slutsky matrix and price elasticities estimates, their standard-errors and t-statistics of the unconstrained static, AR and PA models (in relative prices)

Unconstrained estimation (relative prices)

	R^2			Adjusted $\bar{R}^2$		
	ST	**AR**	**PA**	**ST**	**AR**	**PA**
Germany	0.62	0.67	0.64	0.59	0.60	0.56
France	0.71	0.73	0.71	0.68	0.66	0.64
Italy	0.32	0.47	0.31	0.25	0.35	0.14
USA	0.78	0.84	0.75	0.76	0.80	0.69

Homogeneous estimation

	R^2			Adjusted $\bar{R}^2$		
	ST	**AR**	**PA**	**ST**	**AR**	**PA**
Germany	0.51	0.58	0.51	0.47	0.49	0.40
France	0.68	0.71	0.68	0.66	0.65	0.61
Italy	0.23	0.45	0.23	0.18	0.33	0.07
USA	0.50	0.64	0.42	0.46	0.56	0.30

Table 5.20: Single equation R^2 and adjusted $\bar{R}^2$ of the unconstrained (in relative prices) and homogeneous models

the clarity and amplitude of the rejection, and the fact that the introduction of a dynamic specification does not produce large differences. The inadequacy of data, of the sample size, of the dynamic specification or of the lag order could explain these results.

5.3.2 Homogeneity and Symmetry

Symmetry was imposed, on the homogeneous model, introducing a selection matrix D_s such that $\text{vec}\,\Pi^* = D_s\pi_s$, where π_s is a vector regrouping the lower triangle price coefficients of Π^* (the coefficient matrix of the homogeneous model). Partitioning this latter as $\Pi^* = \begin{pmatrix} \Pi_1^* & \Pi_2^* \end{pmatrix}$, with:

$$\Pi_1^* = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix} \qquad \text{and} \qquad \Pi_2^* = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix},$$

and defining Π_{1s}^* as the symmetric and homogeneous price coefficients matrix:

$$\Pi_{1s}^* = \begin{pmatrix} \gamma_{11} & \gamma_{21} & \gamma_{31} \\ \gamma_{21} & \gamma_{22} & \gamma_{32} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix},$$

the null hypothesis of the symmetry (and homogeneity) test (against the alternative of homogeneity) can be expressed as $H_0 : \Pi_1^* = \Pi_{1s}^*$, the alternative as $H_1 : \Pi_1^* \neq \Pi_{1s}^*$. The results of this test are presented in Table 5.21. The LR and adjusted-LR (using Italianer's small sample correction) test statistics follow a χ_3^2 distribution.

	L_{H_0}	L_{H_1}	LR stat	Adj LR stat	$\chi^2_{3,0.95}$	$\chi^2_{3,0.99}$
Static	1529.24	1544.35	30.23	28.31		
Autoregressive-errors	1581.43	1597.62	32.38	26.82	7.82	11.34
Partial-adjustment	1587.20	1590.64	6.90	5.71		

Table 5.21: Likelihood ratio (LR) statistics for the symmetry test (against homogeneity)

The null hypothesis can not be rejected (at both 5% and 1%) for the partial-adjustment model. We may not then, exclude the presence of symmetry for this model. Symmetry is, however, rejected for the static and autoregressive-errors models. We only find partial support for the conclusion we drew in the previous section when comparing the estimation results under symmetry and under homogeneity. We saw that imposing symmetry had a negligible impact on the estimates and we interpreted this result as a signal of the justification of the symmetry restrictions.

Symmetry can also be tested against the unconstrained model. Although, the result is obvious: if we rejected the homogeneous model, we will reject the more restricted symmetric and homogeneous model. The null hypothesis of this test will combine the null hypotheses of the homogeneity and the symmetry tests. When we tested homogeneity, we partitioned the (unconstrained) matrix of coefficients as $\Pi^h = \begin{pmatrix}\Pi^{**} & \Pi^*\end{pmatrix}$. When we tested symmetry, the sub-matrix Π^* has been furthermore divided in the matrix of price coefficients Π_1^* and the vector of real expenditure coefficients Π_2^*. The null hypothesis of the homogeneous and symmetric model test against the unconstrained model can then be expressed as $H_0 : \begin{pmatrix}\Pi^{**} & \Pi_1^*\end{pmatrix} = \begin{pmatrix}0_{3\times 1} & \Pi_{1s}^*\end{pmatrix}$, where Π_{1s}^* denotes the symmetric price coefficients matrix and $0_{3\times 1}$ a 3×1 null vector, against the alternative $H_1 : \begin{pmatrix}\Pi^{**} & \Pi_1^*\end{pmatrix} \neq \begin{pmatrix}0_{3\times 1} & \Pi_{1s}^*\end{pmatrix}$. For this test, the LR and adjusted LR (using Italianer's correction factor) test statistics will follow a χ_6^2 distribution.

	L_{H_0}	L_{H_1}	LR stat	Adj LR stat	$\chi^2_{6,0.95}$	$\chi^2_{6,0.99}$
Static	1529.24	1614.73	170.98	158.62		
Autoregressive-errors	1581.43	1662.92	162.99	133.53	12.59	16.81
Partial-adjustment	1587.20	1654.55	134.72	110.37		

Table 5.22: Likelihood ratio (LR) statistics for the symmetry and homogeneity test

The results of Table 5.22 are clear – the null hypothesis is strongly rejected for all dynamic models. Again, the impact of the dynamic specifications is small and does not affect the test results.

5.3.3 Dynamic Specification

The second set of hypotheses we will test concerns the dynamic specification of the models. In Section 2.3, we showed that the static, autoregressive-errors and partial-adjustment models are restricted forms of the error-correction model. We also mentioned that with $p = 2$ and $q = 0$, the error-correction model reduces to the partial-adjustment model (the models and the restrictions we imposed are summarized in Table 5.1):

Static Model: The static model:

$$\triangle_s w_t = \Pi \triangle_s x_t + u_t,$$

with $\Pi = \Gamma_0$, is the most restricted form of the error-correction model. It is obtained from this latter by setting $\Phi_i = 0$ and $\Gamma_i = 0$ for $i > 0$. The static model is also a restricted form of the autoregressive-errors and partial-adjustment models (by setting $\Phi_i = 0$ for $i > 0$).

Autoregressive-errors Model: It has been defined as:

$$\triangle_s w_t = \Gamma_0 \triangle_s x_t + v_t$$

with $v_t = \Phi_1 v_{t-1} + \Phi_2 v_{t-2} + u_t$ and it is derived from the error-correction model by imposing the common factor restrictions $\Gamma(L) = \Phi(L)\Gamma_0$. With $p = 2$, these restrictions are $\Gamma_1 = -\Phi_1\Gamma_0$ and $\Gamma_2 = -\Phi_2\Gamma_0$. Obviously, testing these restrictions against the general dynamic model requires that the lag lengths of the endogenous and exogenous variables polynomials are equal (i.e. that $p = q$). Otherwise, and it is our case, the autoregressive-errors model can not be tested against the general dynamic model (it is no more a restricted form of this one).

Partial-adjustment Model: In our particular case, with $p = 2$ and $q = 0$, the error-correction model reduces to the partial-adjustment model and it can be written as:

$$\Phi(L)\triangle_s w_t = \Phi(1)\Pi\triangle_s x_t + u_t$$

where $\Pi = \Phi^{-1}(1)\Gamma_0$ and $\Phi(L) = I - \Phi_1 L - \Phi_2 L$.

Summarizing, given the chosen lag specification ($p = 2$ and $q = 0$), we can only test the static model against the autoregressive-errors and partial-adjustment models. In practice, we test the necessity of a dynamic specification.

The null hypothesis for these tests is $H_0 : \begin{pmatrix}\Phi_1 & \Phi_2\end{pmatrix} = \begin{pmatrix}0 & 0\end{pmatrix}$ against the alternative $H_1 : \begin{pmatrix}\Phi_1 & \Phi_2\end{pmatrix} \neq \begin{pmatrix}0 & 0\end{pmatrix}$. The number of imposed restrictions is 18 and the LR (like the adjusted LR since $q_1 = 6$ and $g = 3$) test statistic will follow a χ^2_{18} distribution.

As seen in Table 5.23, the static model is always strongly rejected against the autoregressive-errors and partial-adjustment models – under all economic restrictions. This result is not surprising. We already mentioned its restrictness, the lack of ability to capture the data dynamics and its serial correlation problems. A dynamic specification seems then, to be necessary.

It would be interesting to check which one of the proposed dynamic models is more appropriate for our data. Unfortunately, with the chosen lag length (justified by the diagnostic and hypothesis tests computed on the ordinary least square residuals of the general dynamic model – see Section 4.5) this is not possible.

5.3.4 Summary

We will conclude this section with a brief summary of the main results we obtained.

Unconstrained Estimation						
H_1	L_{H_0}	L_{H_1}	LR stat	Adj LR stat	$\chi^2_{18,0.95}$	$\chi^2_{18,0.99}$
Autoregressive-errors	1614.73	1662.92	96.39	90.58	28.87	34.81
Partial-adjustment		1654.55	79.64	74.85		

Homogeneous Estimation						
H_1	L_{H_0}	L_{H_1}	LR stat	Adj LR stat	$\chi^2_{18,0.95}$	$\chi^2_{18,0.99}$
Autoregressive-errors	1544.35	1597.62	106.53	100.76	28.87	34.81
Partial-adjustment		1590.64	92.58	87.56		

Symmetric and Homogeneous Estimation						
H_1	L_{H_0}	L_{H_1}	LR stat	Adj LR stat	$\chi^2_{18,0.95}$	$\chi^2_{18,0.99}$
Autoregressive-errors	1529.24	1581.43	104.38	98.72	28.87	34.81
Partial-adjustment		1587.20	115.91	109.63		

Table 5.23: Likelihood ratio (LR) test statistics for the test of the static models against the autoregressive-errors and partial-adjustment models

1. **Homogeneity:** The results are similar for all the models: the homogeneity hypothesis is strongly rejected. The results of the LR test statistics are confirmed by the significance tests of the omitted variable in the individual equations and, for the static model, by the Laitinen F-test. This is surprising, on the one hand for the clarity of the rejection, and, on the other hand, the fact that the dynamic specification has no impact on the test conclusions. Thus, our results are in contradiction with the literature that has often explained the rejection of homogeneity with the lack of dynamics. The inadequacies of data, the sample size, the dynamic specification or the chosen lag order could represent an explanation.

2. **Symmetry:** The symmetry restrictions are not rejected for the partial-adjustment model when tested against the homogeneous model. The results partially confirm the conclusions drawn in Section 5.2: symmetry seems to be justified by data.

3. **Dynamic specification:** The chosen lag specifications only allow the test of the static model against the autoregressive-errors and partial-adjustment models. The static model is, as expected, clearly rejected against all the alternative dynamic specifications giving evidence of the necessity of a dynamic model (the economic restrictions have no impact on the test results). Unfortunately, we can not test the autoregressive-errors model against the general dynamic model and thus we can not determine which dynamic specification is the best one for our data.

Of course, the results we obtained depend on the chosen data and lag specifications. We justified our choices in Chapter 4. It would be interesting, however, to check if the results would change with different values of p and q (in particular with $p = q$ which would allow to test the autoregressive-errors model against the error-correction model) or by using data in first-differences (that ensured joint-stationarity, too). We leave this topic open to further research.

Chapter 6

Conclusions

In this work, we studied the homogeneity and symmetry tests in the context of a dynamic import allocation model. Several dynamic specifications have been considered, from the general dynamic model to the more restricted autoregressive-errors, partial-adjustment and static models. For all of them, unconstrained and constrained (under homogeneity and symmetry) estimates were computed in order to compare the model performances and to test the economic and dynamic restrictions.

We started our analysis by presenting the static import allocation model introduced by Winters (1984). Winters was the first to choose the AID (Almost Ideal Demand System) model as a functional form. Despite the advantages of his choice, he concluded his study rejecting, on the one hand, the homogeneity and symmetry restrictions and, on the other hand, the usual, in the import allocation context, separability assumption between foreign and domestic sources. Winters identified the neglect of dynamics as one of the reasons of the rejection, in addition to the data inadequacy. Our aim was to investigate, using Swiss data, if the introduction of dynamics produces different results, especially with what concerns the homogeneity and symmetry tests.

For this purpose, we presented some extensions to the static AID import allocation model. We first introduced a general dynamic model and, in particular, its error-correction form. We showed that, from the error-correction model, we can obtain the autoregressive-errors, partial-adjustment and static models by imposing restrictions on the lag polynomials of the endogenous and exogenous variables. We also showed what the implications of the adding-up, homogeneity and symmetry restrictions are and how to estimate the unconstrained and constrained models using maximum-likelihood techniques.

The empirical part of the work was carried out using monthly Swiss data taken from the Swiss-Impex database (source: Swiss Federal Custom Autority). The choice of seasonal-differences was justified by the requirement of (joint) stationarity in order to make the equilibrium multipliers estimable. In fact, preliminary analysis of data in levels revealed the presence of non-stationarity and seasonality.

The unconstrained estimates highlighted plausibility problems, with negative cross-price Slutsky coefficients, cross-prices elasticities and expenditure elasticities. The introduction of homogeneity seemed to correct these problems, thus increasing the results plausibility. However, homogeneity was strongly rejected, either when tested in the overall system with likelihood ratio statistics, or when tested in the individual equations with a t-test. The test results are similar across the models, thus giving evidence that the dynamic specification has no (or little) influence. The presence of unit roots, the inadequacies of data, the sample size, or the chosen lag length could explain our results.

Symmetry had a minor impact on the coefficient estimates, but it strongly reduced their asymptotic standard-errors. Unconstrained and homogeneous estimations were characterized by a similar magnitude of results, in particular to what concerns the symmetric cross-price elements. This evidence of symmetry was only partly confirmed by hypothesis testing: symmetry was not rejected for the partial-adjustment model.

We also investigated the impact of the different dynamic specifications. In the literature, the static model has often been discarded because of its restrictiveness, its inability to capture the data dynamics and its serial correlation problems. Compared to the autoregressive-errors or partial-adjustment models, the performances of the static model are of worse quality. Goodness-of-fit or information criteria suggest choosing one of the dynamic models, and, in addition, residual analysis revealed the presence of serial correlation in the static residuals. The fact that the static model is strongly rejected against the autoregressive-errors and partial-adjustment models is not surprising. But, as we mentioned before, if the dynamic specification has an impact on the model performances, it has no influence on the homogeneity and symmetry test conclusions.

Hence, we could not prove that the dynamic mis-specification is the main reason that leads to the rejection of the regularity restrictions, as highlighted for example by Winters (1984). Inadequacies of the data or the limited sample size could have an impact, but to our knowledge no studies on import functions have been carried out before using Swiss data. The data and lag specifications probably also have an influence. We proposed a procedure to justify their choice – several alternatives can be considered. We can not exclude that the use of first-differences or the choice of a different lag length of the matrix polynomials provides different results, nor we can exclude the presence of unit roots in seasonal differences. A comparison between these different alternatives could be a topic for further research.

Appendix A

Standard International Trade Classification (Rev. 3)

SITC Rev.3, sections 5 - 8, 2 digits level

5 Chemicals and related products, n.e.s.

51 Organic chemicals

52 Inorganic chemicals

53 Dyeing, tanning and colouring materials

54 Medicinal and pharmaceutical products

55 Essential oils and resinoids and perfume materials; toilet, polishing and cleansing preparations

56 Fertilizers (other than those of group 272)

57 Plastics in primary forms

58 Plastics in non-primary forms

59 Chemical materials and products, n.e.s.

6 Manufactured goods classified chiefly by material

- 61 Leather, leather manufactures, n.e.s., and dressed furskins
- 62 Rubber manufactures, n.e.s.
- 63 Cork and wood manufactures (excluding furniture)
- 64 Paper, paperboard and articles of paper pulp, of paper or of paperboard
- 65 Textile yarn, fabrics, made-up articles, n.e.s., and related products
- 66 Non-metallic mineral manufactures, n.e.s.
- 67 Iron and steel
- 68 Non-ferrous metals
- 69 Manufactures of metals, n.e.s.

7 Machinery and transport equipment

- 71 Power-generating machinery and equipment
- 72 Machinery specialized for particular industries
- 73 Metalworking machinery
- 74 General industrial machinery and equipment, n.e.s., and machine parts, n.e.s.
- 75 Office machines and automatic data-processing machines
- 76 Telecommunications and sound-recording and reproducing apparatus and equipment
- 77 Electrical machinery, apparatus and appliances, n.e.s., and electrical parts thereof (including non-electrical counterparts, n.e.s., of electrical household-type equipment)

78 Road vehicles (including air-cushion vehicles)

79 Other transport equipment

8 Miscellaneous manufactured articles

81 Prefabricated buildings; sanitary, plumbing, heating and lighting fixtures and fittings, n.e.s.

82 Furniture, and parts thereof; bedding, mattresses, mattress supports, cushions and similar stuffed furnishings

83 Travel goods, handbags and similar containers

84 Articles of apparel and clothing accessories

85 Footwear

87 Professional, scientific and controlling instruments and apparatus, n.e.s.

88 Photographic apparatus, equipment and supplies and optical goods, n.e.s.; watches and clocks

89 Miscellaneous manufactured articles, n.e.s.

Source: United Nations Statistics Division (http://unstats.un.org)

Appendix B

Descriptive Statistics

This appendix contains some descriptive statistics of the different data specifications for each of the variables of the model, i.e. the logarithms of budget shares (w_{it}) and import prices (p_{it}) for Germany, France, Italy and USA, and the real expenditure term ($\log \frac{M_t}{P_t}$ where M_t is the total expenditure on imports and P_t is a Stone import price index). The first two columns concerns data in levels, the third and fourth first-differences (defined as $\triangle z_t = z_t - z_{t-1}$) and the last two columns seasonal differences (defined as $\triangle_s z_t = z_t - z_{t-12}$). Seasonal adjusted data (denoted by " sa ") are computed using the US Census Bureau's X12 program, implemented with the Eviews 4.1 software.

	German budget shares					
	w_{it}	w_{it}^{sa}	$\triangle w_{it}$	$\triangle w_{it}^{sa}$	$\triangle_s w_{it}$	$\triangle_s w_{it}^{sa}$
Mean	0.5602	0.5603	0.0001	0.0002	-0.0017	-0.0017
Median	0.5589	0.5622	-0.0022	-0.0001	-0.0031	-0.0029
Maximum	0.6354	0.6082	0.1005	0.0634	0.0595	0.0582
Minimum	0.5013	0.5058	-0.0708	-0.0337	-0.0652	-0.0657
Std. Dev.	0.0256	0.0191	0.0279	0.0157	0.0223	0.0222
Skewness	0.3075	-0.2556	0.5269	0.6930	0.0973	0.0813
Kurtosis	2.9456	2.9609	4.0918	5.1158	3.4499	3.4302
Jarque-Bera	2.8584	1.9709	17.1738	47.7142	1.6822	1.4805
Probability	0.2395	0.3733	0.0002	0.0000	0.4312	0.4770
Observations	180	180	179	179	168	168
	French budget shares					
	w_{it}	w_{it}^{sa}	$\triangle w_{it}$	$\triangle w_{it}^{sa}$	$\triangle_s w_{it}$	$\triangle_s w_{it}^{sa}$
Mean	0.1656	0.1656	0.0000	0.0000	0.0001	0.0001
Median	0.1632	0.1634	-0.0010	-0.0004	0.0002	0.0006
Maximum	0.2200	0.2114	0.0660	0.0547	0.0460	0.0468
Minimum	0.1320	0.1312	-0.0586	-0.0458	-0.0501	-0.0558
Std. Dev.	0.0148	0.0129	0.0188	0.0153	0.0180	0.0180
Skewness	0.6882	1.0678	0.1291	0.1525	-0.0722	-0.1198
Kurtosis	3.6873	4.5958	4.1554	4.6337	3.3106	3.4362
Jarque-Bera	17.7492	53.3060	10.4540	20.6002	0.8212	1.7340
Probability	0.0001	0.0000	0.0054	0.0000	0.6632	0.4202
Observations	180	180	179	179	168	168
	Italian budget shares					
	w_{it}	w_{it}^{sa}	$\triangle w_{it}$	$\triangle w_{it}^{sa}$	$\triangle_s w_{it}$	$\triangle_s w_{it}^{sa}$
Mean	0.1598	0.1600	0.0002	0.0002	0.0008	0.0008
Median	0.1605	0.1591	0.0016	-0.0002	0.0011	0.0010
Maximum	0.1989	0.1867	0.0524	0.0189	0.0234	0.0232
Minimum	0.1046	0.1423	-0.0854	-0.0200	-0.0275	-0.0290
Std. Dev.	0.0167	0.0072	0.0263	0.0079	0.0099	0.0101
Skewness	-0.6858	0.6320	-1.2745	0.0323	-0.0874	-0.1093
Kurtosis	4.6884	4.1773	5.3654	2.9730	2.8216	2.9662
Jarque-Bera	35.4882	22.3777	90.1947	0.0366	0.4365	0.3422
Probability	0.0000	0.0000	0.0000	0.9819	0.8039	0.8427
Observations	180	180	179	179	168	168

	US budget shares					
	w_{it}	w_{it}^{sa}	$\triangle w_{it}$	$\triangle w_{it}^{sa}$	$\triangle_s w_{it}$	$\triangle_s w_{it}^{sa}$
Mean	0.1139	0.1139	0.0002	0.0002	0.0014	0.0013
Median	0.1108	0.1123	0.0007	0.0007	0.0012	0.0028
Maximum	0.2009	0.1904	0.0826	0.0420	0.0737	0.0803
Minimum	0.0739	0.0787	-0.0728	-0.0508	-0.0860	-0.0933
Std. Dev.	0.0207	0.0177	0.0237	0.0164	0.0251	0.0250
Skewness	0.8975	0.9099	-0.0571	-0.2264	-0.1751	-0.2442
Kurtosis	4.4053	5.1220	3.7521	3.7897	3.7379	4.0173
Jarque-Bera	38.9780	58.6121	4.3166	6.1809	4.6705	8.9127
Probability	0.0000	0.0000	0.1155	0.0455	0.0968	0.0116
Observations	180	180	179	179	168	168
	German import prices					
	p_{it}	p_{it}^{sa}	$\triangle p_{it}$	$\triangle p_{it}^{sa}$	$\triangle_s p_{it}$	$\triangle_s p_{it}^{sa}$
Mean	3.2632	3.2627	-0.0002	-0.0009	0.0080	0.0080
Median	3.2239	3.2283	-0.0340	-0.0224	0.0322	0.0424
Maximum	3.9651	3.9491	0.7541	0.7922	0.6312	0.6260
Minimum	2.9092	2.9587	-0.4922	-0.4923	-0.6062	-0.6339
Std. Dev.	0.1931	0.1882	0.2081	0.2009	0.2511	0.2522
Skewness	0.5711	0.5919	0.9753	0.8341	-0.2672	-0.2814
Kurtosis	2.9120	2.9764	4.4716	4.5879	2.5006	2.5087
Jarque-Bera	9.8425	10.5130	44.5297	39.5622	3.7441	3.9066
Probability	0.0073	0.0052	0.0000	0.0000	0.1538	0.1418
Observations	180	180	179	179	168	168
	French import prices					
	p_{it}	p_{it}^{sa}	$\triangle p_{it}$	$\triangle p_{it}^{sa}$	$\triangle_s p_{it}$	$\triangle_s p_{it}^{sa}$
Mean	3.6450	3.6437	0.0053	0.0026	0.0263	0.0260
Median	3.5381	3.5442	-0.0050	0.0195	-0.0078	-0.0088
Maximum	5.4924	5.4802	1.9884	1.9502	1.8381	1.8288
Minimum	3.1514	3.0476	-1.9372	-1.9912	-1.9983	-1.9838
Std. Dev.	0.3882	0.3619	0.4978	0.4600	0.4474	0.4462
Skewness	1.7948	1.9547	-0.1236	-0.3094	-0.0092	-0.0190
Kurtosis	7.3651	8.7351	7.7056	9.9360	6.3057	6.3114
Jarque-Bera	239.5506	361.3187	165.6038	361.6662	76.4959	76.7672
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations	180	180	179	179	168	168

	Italian import prices					
	p_{it}	p_{it}^{sa}	$\triangle p_{it}$	$\triangle p_{it}^{sa}$	$\triangle_s p_{it}$	$\triangle_s p_{it}^{sa}$
Mean	3.3400	3.3402	0.0057	0.0048	0.0456	0.0455
Median	3.3248	3.3336	0.0199	0.0033	0.0389	0.0394
Maximum	4.0897	3.9802	0.3467	0.2989	0.4557	0.4431
Minimum	2.8794	2.9025	-0.4067	-0.3592	-0.4378	-0.4318
Std. Dev.	0.2065	0.1885	0.1440	0.1008	0.1509	0.1508
Skewness	0.5878	0.7338	-0.4094	-0.1850	0.1004	0.0831
Kurtosis	4.0708	3.8904	2.9800	4.7725	3.3253	3.2837
Jarque-Bera	18.9629	22.0993	5.0045	24.4528	1.0227	0.7569
Probability	0.0001	0.0000	0.0819	0.0000	0.5997	0.6849
Observations	180	180	179	179	168	168
	US import prices					
	p_{it}	p_{it}^{sa}	$\triangle p_{it}$	$\triangle p_{it}^{sa}$	$\triangle_s p_{it}$	$\triangle_s p_{it}^{sa}$
Mean	6.2901	6.2884	0.0060	0.0040	0.0873	0.0863
Median	6.2143	6.2367	-0.0019	0.0443	0.0934	0.1127
Maximum	8.5515	8.3633	1.7773	1.6476	1.6843	1.7576
Minimum	5.0010	5.1032	-1.7308	-1.9266	-2.4091	-2.3676
Std. Dev.	0.6145	0.5523	0.7336	0.6379	0.7445	0.7405
Skewness	0.7317	0.6462	-0.1274	-0.1058	-0.2063	-0.2317
Kurtosis	3.7760	3.7378	2.7254	2.8234	3.2538	3.1966
Jarque-Bera	20.5763	16.6099	1.0470	0.5665	1.6420	1.7742
Probability	0.0000	0.0002	0.5925	0.7533	0.4400	0.4119
Observations	180	180	179	179	168	168
	Real expenditure term					
	$\log \frac{M_t}{P_t}$	$\log \frac{M_t^{sa}}{P_t^{sa}}$	$\triangle \log \frac{M_t}{P_t}$	$\triangle \log \frac{M_t^{sa}}{P_t^{sa}}$	$\triangle_s \log \frac{M_t}{P_t}$	$\triangle_s \log \frac{M_t^{sa}}{P_t^{sa}}$
Mean	5.9684	5.9704	-0.0011	0.0003	-0.0169	-0.0168
Median	5.9826	5.9800	0.0081	-0.0038	-0.0109	-0.0059
Maximum	6.4539	6.4019	0.4596	0.4384	0.6262	0.6308
Minimum	5.5345	5.5075	-0.5530	-0.5913	-0.5012	-0.5167
Std. Dev.	0.1684	0.1538	0.1994	0.1706	0.2176	0.2178
Skewness	-0.2658	-0.1437	-0.2848	-0.3934	0.1411	0.1385
Kurtosis	3.1731	3.3834	2.8269	4.4048	2.7237	2.7667
Jarque-Bera	2.3448	1.7226	2.6435	19.3341	1.0917	0.9178
Probability	0.3096	0.4226	0.2667	0.0001	0.5793	0.6320
Observations	180	180	179	179	168	168

Appendix C

Data

The following tables resume data on the import budget shares and import prices for Germany, France, Italy and USA, and on real expenditure from January 1989 to December 2002. Data in levels are computed from the Swiss-Impex database of the Swiss Federal Customs Administration. Seasonal differences, used for the estimation of the different models presented in Section 2.3, are defined as $\triangle_s z_t = z_t - z_{t-12}$.

Import prices, for each country, are computed as Stone's price indexes using data at the lower, and more detailed, level (5 digits) of the SITC classification, defining expenditures shares on "items" as weights and the logarithm of their unit values as prices.

The logarithm of real expenditure is computed as the difference between the logarithm of total expenditure on imports (the sum of imports from the four countries) at time t, M_t, and a Stone price index defined as $\log P_t = \sum_{k=1}^{4} w_{kt} p_{kt}$ where w_{kt} denotes the budget share and p_{kt} the import price of country k at time t (or equivalently the logarithm of the ratio M_t/P_t).

Import budget shares from Germany, France, Italy and USA (by month, 1989-2002)

Year	Levels				Seasonal differences			
	Germany	France	Italy	USA	Germany	France	Italy	USA
01:1989	0.5690	0.1627	0.1509	0.1175	0.0396	0.0016	0.0105	0.0349
02:1989	0.5669	0.1845	0.1593	0.0893	-0.0203	0.0320	-0.0050	-0.0067
03:1989	0.5735	0.1561	0.1734	0.0970	-0.0354	-0.0024	0.0148	0.0231
04:1989	0.5638	0.1522	0.1528	0.1312	-0.0329	0.0007	-0.0018	0.0340
05:1989	0.5570	0.1488	0.1757	0.1185	-0.0239	-0.0036	0.0093	0.0182
06:1989	0.5641	0.1607	0.1574	0.1179	0.0101	-0.0036	-0.0142	0.0077
07:1989	0.5547	0.1620	0.1956	0.0877	-0.0089	0.0137	0.0018	-0.0066
08:1989	0.6149	0.1328	0.1205	0.1318	-0.0094	-0.0184	-0.0053	0.0331
09:1989	0.5858	0.1492	0.1575	0.1075	-0.0166	-0.0069	0.0012	0.0223
10:1989	0.5817	0.1485	0.1663	0.1035	-0.0121	-0.0088	-0.0057	0.0265
11:1989	0.5726	0.1560	0.1655	0.1059	-0.0159	0.0006	0.0020	0.0133
12:1989	0.5770	0.1694	0.1605	0.0931	-0.0183	0.0238	0.0028	-0.0082
01:1990	0.5809	0.1580	0.1505	0.1105	0.0119	-0.0046	-0.0003	-0.0069
02:1990	0.5749	0.1844	0.1560	0.0846	0.0080	-0.0001	-0.0032	-0.0047
03:1990	0.5753	0.1633	0.1662	0.0951	0.0018	0.0073	-0.0072	-0.0018
04:1990	0.5726	0.1571	0.1732	0.0971	0.0087	0.0048	0.0204	-0.0340
05:1990	0.5470	0.1519	0.1694	0.1317	-0.0100	0.0031	-0.0064	0.0133
06:1990	0.5364	0.1602	0.1782	0.1252	-0.0277	-0.0005	0.0209	0.0073
07:1990	0.5402	0.1581	0.1928	0.1090	-0.0145	-0.0040	-0.0028	0.0213
08:1990	0.6216	0.1466	0.1248	0.1070	0.0067	0.0138	0.0043	-0.0248
09:1990	0.5961	0.1566	0.1632	0.0841	0.0103	0.0074	0.0057	-0.0234
10:1990	0.5743	0.1757	0.1686	0.0815	-0.0074	0.0271	0.0023	-0.0220
11:1990	0.5855	0.1638	0.1617	0.0890	0.0129	0.0077	-0.0037	-0.0169
12:1990	0.5840	0.1549	0.1729	0.0882	0.0070	-0.0145	0.0123	-0.0049
01:1991	0.5781	0.1709	0.1581	0.0929	-0.0029	0.0128	0.0076	-0.0176
02:1991	0.5556	0.1878	0.1555	0.1011	-0.0194	0.0034	-0.0005	0.0165
03:1991	0.5618	0.1585	0.1585	0.1212	-0.0135	-0.0049	-0.0077	0.0261
04:1991	0.5344	0.1662	0.1457	0.1537	-0.0381	0.0091	-0.0275	0.0565
05:1991	0.5540	0.1459	0.1607	0.1395	0.0070	-0.0061	-0.0087	0.0078
06:1991	0.5331	0.1514	0.1524	0.1631	-0.0033	-0.0088	-0.0258	0.0379
07:1991	0.5486	0.1597	0.1911	0.1007	0.0084	0.0016	-0.0017	-0.0083
08:1991	0.6071	0.1320	0.1091	0.1518	-0.0145	-0.0147	-0.0157	0.0448
09:1991	0.5678	0.1569	0.1529	0.1223	-0.0283	0.0003	-0.0102	0.0382
10:1991	0.5562	0.1628	0.1604	0.1206	-0.0181	-0.0129	-0.0081	0.0392
11:1991	0.5634	0.1529	0.1579	0.1257	-0.0221	-0.0108	-0.0038	0.0367
12:1991	0.5724	0.1674	0.1576	0.1027	-0.0117	0.0125	-0.0153	0.0145
01:1992	0.5751	0.1695	0.1533	0.1020	-0.0030	-0.0014	-0.0048	0.0091
02:1992	0.5666	0.1574	0.1583	0.1177	0.0110	-0.0304	0.0028	0.0166
03:1992	0.5695	0.1576	0.1596	0.1134	0.0076	-0.0009	0.0011	-0.0078
04:1992	0.5429	0.1560	0.1542	0.1470	0.0085	-0.0102	0.0085	-0.0067
05:1992	0.5620	0.1918	0.1503	0.0959	0.0080	0.0460	-0.0104	-0.0436
06:1992	0.5331	0.1612	0.1721	0.1337	-0.0001	0.0098	0.0196	-0.0293
07:1992	0.5714	0.1481	0.1989	0.0816	0.0229	-0.0116	0.0078	-0.0191

Year	Levels				Seasonal differences			
	Germany	France	Italy	USA	Germany	France	Italy	USA
08:1992	0.6181	0.1527	0.1135	0.1158	0.0109	0.0207	0.0044	-0.0360
09:1992	0.6097	0.1543	0.1474	0.0886	0.0419	-0.0026	-0.0056	-0.0337
10:1992	0.5913	0.1654	0.1539	0.0893	0.0351	0.0027	-0.0065	-0.0313
11:1992	0.5646	0.1665	0.1609	0.1080	0.0012	0.0136	0.0029	-0.0177
12:1992	0.5835	0.1521	0.1549	0.1095	0.0112	-0.0153	-0.0027	0.0068
01:1993	0.5670	0.1764	0.1403	0.1163	-0.0082	0.0069	-0.0130	0.0143
02:1993	0.5672	0.1673	0.1588	0.1067	0.0006	0.0099	0.0005	-0.0111
03:1993	0.5798	0.1618	0.1568	0.1015	0.0104	0.0042	-0.0028	-0.0118
04:1993	0.5600	0.1702	0.1485	0.1213	0.0171	0.0142	-0.0057	-0.0257
05:1993	0.5738	0.1664	0.1643	0.0955	0.0118	-0.0254	0.0140	-0.0004
06:1993	0.5281	0.1985	0.1668	0.1066	-0.0050	0.0373	-0.0052	-0.0271
07:1993	0.5589	0.1552	0.1904	0.0955	-0.0126	0.0071	-0.0085	0.0139
08:1993	0.6015	0.1549	0.1265	0.1171	-0.0166	0.0022	0.0130	0.0014
09:1993	0.5829	0.1620	0.1502	0.1049	-0.0267	0.0077	0.0028	0.0163
10:1993	0.5715	0.1690	0.1593	0.1002	-0.0197	0.0036	0.0053	0.0108
11:1993	0.5542	0.1680	0.1633	0.1144	-0.0104	0.0015	0.0025	0.0064
12:1993	0.5782	0.1504	0.1505	0.1209	-0.0053	-0.0017	-0.0045	0.0115
01:1994	0.5404	0.1838	0.1500	0.1258	-0.0266	0.0074	0.0097	0.0095
02:1994	0.5543	0.1769	0.1647	0.1042	-0.0130	0.0096	0.0059	-0.0025
03:1994	0.5641	0.1732	0.1614	0.1013	-0.0157	0.0114	0.0046	-0.0002
04:1994	0.5572	0.1740	0.1539	0.1149	-0.0028	0.0038	0.0054	-0.0064
05:1994	0.5626	0.1669	0.1652	0.1054	-0.0112	0.0005	0.0009	0.0098
06:1994	0.5679	0.1632	0.1633	0.1056	0.0399	-0.0353	-0.0035	-0.0010
07:1994	0.5550	0.1618	0.1866	0.0966	-0.0039	0.0066	-0.0038	0.0011
08:1994	0.6003	0.1448	0.1193	0.1356	-0.0012	-0.0101	-0.0072	0.0185
09:1994	0.5918	0.1653	0.1489	0.0940	0.0089	0.0033	-0.0012	-0.0109
10:1994	0.5628	0.1838	0.1650	0.0884	-0.0088	0.0147	0.0057	-0.0117
11:1994	0.5822	0.1626	0.1533	0.1020	0.0279	-0.0054	-0.0101	-0.0125
12:1994	0.5589	0.1696	0.1658	0.1057	-0.0193	0.0192	0.0154	-0.0153
01:1995	0.5546	0.1707	0.1515	0.1232	0.0142	-0.0131	0.0015	-0.0026
02:1995	0.5904	0.1646	0.1623	0.0828	0.0361	-0.0123	-0.0024	-0.0214
03:1995	0.5959	0.1548	0.1539	0.0954	0.0317	-0.0183	-0.0075	-0.0059
04:1995	0.5652	0.1950	0.1457	0.0941	0.0080	0.0209	-0.0081	-0.0208
05:1995	0.5773	0.1575	0.1621	0.1031	0.0147	-0.0093	-0.0031	-0.0022
06:1995	0.5345	0.1805	0.1707	0.1143	-0.0334	0.0173	0.0075	0.0087
07:1995	0.5796	0.1606	0.1847	0.0751	0.0246	-0.0012	-0.0020	-0.0214
08:1995	0.6171	0.1443	0.1283	0.1103	0.0168	-0.0005	0.0090	-0.0253
09:1995	0.5625	0.1806	0.1560	0.1008	-0.0293	0.0153	0.0071	0.0069
10:1995	0.5438	0.1857	0.1624	0.1081	-0.0189	0.0019	-0.0026	0.0197
11:1995	0.5560	0.1809	0.1520	0.1110	-0.0262	0.0183	-0.0012	0.0091
12:1995	0.5463	0.1775	0.1537	0.1226	-0.0126	0.0079	-0.0122	0.0169
01:1996	0.5428	0.1896	0.1554	0.1121	-0.0117	0.0189	0.0039	-0.0111
02:1996	0.5485	0.1732	0.1715	0.1068	-0.0419	0.0086	0.0092	0.0241

Year	Levels				Seasonal differences			
	Germany	France	Italy	USA	Germany	France	Italy	USA
03:1996	0.5387	0.1844	0.1610	0.1160	-0.0572	0.0296	0.0071	0.0206
04:1996	0.5088	0.1832	0.1639	0.1441	-0.0565	-0.0117	0.0181	0.0501
05:1996	0.5249	0.1719	0.1774	0.1259	-0.0524	0.0143	0.0153	0.0227
06:1996	0.5359	0.1667	0.1707	0.1267	0.0015	-0.0138	-0.0001	0.0124
07:1996	0.5144	0.1873	0.1941	0.1042	-0.0652	0.0267	0.0094	0.0291
08:1996	0.5978	0.1472	0.1227	0.1323	-0.0193	0.0029	-0.0056	0.0220
09:1996	0.5429	0.1627	0.1730	0.1213	-0.0196	-0.0179	0.0170	0.0205
10:1996	0.5405	0.1679	0.1719	0.1197	-0.0033	-0.0178	0.0095	0.0116
11:1996	0.5605	0.1622	0.1712	0.1061	0.0045	-0.0188	0.0191	-0.0049
12:1996	0.5013	0.1896	0.1749	0.1342	-0.0450	0.0121	0.0213	0.0116
01:1997	0.5326	0.1869	0.1592	0.1213	-0.0103	-0.0027	0.0038	0.0092
02:1997	0.5292	0.1707	0.1693	0.1308	-0.0193	-0.0024	-0.0022	0.0240
03:1997	0.5328	0.1686	0.1679	0.1307	-0.0059	-0.0158	0.0070	0.0147
04:1997	0.5238	0.1630	0.1476	0.1656	0.0150	-0.0203	-0.0162	0.0215
05:1997	0.5114	0.1769	0.1670	0.1447	-0.0135	0.0051	-0.0104	0.0188
06:1997	0.5183	0.1782	0.1575	0.1459	-0.0176	0.0115	-0.0131	0.0193
07:1997	0.5117	0.1893	0.1806	0.1184	-0.0028	0.0020	-0.0134	0.0142
08:1997	0.5535	0.1410	0.1046	0.2009	-0.0443	-0.0062	-0.0181	0.0686
09:1997	0.5448	0.1701	0.1569	0.1281	0.0019	0.0074	-0.0161	0.0068
10:1997	0.5542	0.1661	0.1592	0.1205	0.0137	-0.0018	-0.0127	0.0008
11:1997	0.5510	0.1528	0.1578	0.1383	-0.0095	-0.0094	-0.0134	0.0322
12:1997	0.5261	0.1664	0.1684	0.1391	0.0248	-0.0231	-0.0065	0.0049
01:1998	0.5276	0.1668	0.1494	0.1562	-0.0049	-0.0202	-0.0098	0.0349
02:1998	0.5466	0.1713	0.1709	0.1113	0.0174	0.0006	0.0016	-0.0196
03:1998	0.5458	0.1632	0.1647	0.1263	0.0130	-0.0054	-0.0033	-0.0043
04:1998	0.5376	0.1791	0.1546	0.1288	0.0137	0.0161	0.0069	-0.0367
05:1998	0.5288	0.1551	0.1570	0.1590	0.0174	-0.0218	-0.0099	0.0143
06:1998	0.5479	0.1675	0.1704	0.1142	0.0296	-0.0107	0.0129	-0.0318
07:1998	0.5582	0.1595	0.1911	0.0912	0.0466	-0.0298	0.0105	-0.0272
08:1998	0.6109	0.1442	0.1175	0.1274	0.0573	0.0033	0.0130	-0.0736
09:1998	0.5401	0.2026	0.1586	0.0988	-0.0047	0.0324	0.0017	-0.0294
10:1998	0.5386	0.1662	0.1595	0.1357	-0.0156	0.0001	0.0003	0.0153
11:1998	0.5713	0.1829	0.1523	0.0935	0.0203	0.0301	-0.0056	-0.0448
12:1998	0.5327	0.1900	0.1477	0.1297	0.0066	0.0235	-0.0207	-0.0094
01:1999	0.5705	0.1626	0.1504	0.1164	0.0428	-0.0041	0.0011	-0.0398
02:1999	0.5449	0.2012	0.1519	0.1019	-0.0016	0.0299	-0.0190	-0.0093
03:1999	0.5367	0.2001	0.1515	0.1117	-0.0091	0.0369	-0.0132	-0.0146
04:1999	0.5401	0.1667	0.1643	0.1288	0.0025	-0.0123	0.0098	0.0000
05:1999	0.5480	0.1739	0.1617	0.1164	0.0192	0.0188	0.0047	-0.0426
06:1999	0.5296	0.2047	0.1602	0.1056	-0.0183	0.0372	-0.0103	-0.0086
07:1999	0.5317	0.1840	0.1795	0.1048	-0.0266	0.0244	-0.0116	0.0137
08:1999	0.5842	0.1858	0.1124	0.1175	-0.0266	0.0416	-0.0051	-0.0099
09:1999	0.5481	0.1938	0.1615	0.0966	0.0081	-0.0087	0.0029	-0.0022

Year	Levels				Seasonal differences			
	Germany	France	Italy	USA	Germany	France	Italy	USA
10:1999	0.5472	0.1946	0.1560	0.1022	0.0086	0.0284	-0.0035	-0.0335
11:1999	0.5666	0.1540	0.1623	0.1170	-0.0047	-0.0289	0.0101	0.0235
12:1999	0.5114	0.2200	0.1515	0.1170	-0.0213	0.0301	0.0038	-0.0126
01:2000	0.5539	0.1615	0.1470	0.1376	-0.0166	-0.0012	-0.0034	0.0212
02:2000	0.5503	0.1602	0.1753	0.1142	0.0054	-0.0411	0.0234	0.0122
03:2000	0.5368	0.1676	0.1570	0.1385	0.0001	-0.0325	0.0056	0.0268
04:2000	0.5519	0.1674	0.1685	0.1122	0.0118	0.0007	0.0041	-0.0167
05:2000	0.5352	0.1524	0.1625	0.1498	-0.0128	-0.0214	0.0008	0.0334
06:2000	0.5093	0.1812	0.1633	0.1462	-0.0203	-0.0235	0.0032	0.0407
07:2000	0.5561	0.1483	0.1916	0.1040	0.0245	-0.0357	0.0121	-0.0009
08:2000	0.5780	0.1375	0.1143	0.1702	-0.0062	-0.0483	0.0019	0.0527
09:2000	0.5396	0.1556	0.1584	0.1464	-0.0085	-0.0382	-0.0031	0.0498
10:2000	0.5146	0.1552	0.1543	0.1759	-0.0325	-0.0394	-0.0017	0.0737
11:2000	0.5208	0.1980	0.1499	0.1312	-0.0458	0.0440	-0.0124	0.0142
12:2000	0.5462	0.1699	0.1551	0.1288	0.0348	-0.0501	0.0036	0.0117
01:2001	0.5657	0.1687	0.1524	0.1132	0.0118	0.0073	0.0053	-0.0244
02:2001	0.5559	0.1753	0.1629	0.1060	0.0055	0.0151	-0.0125	-0.0082
03:2001	0.5521	0.1679	0.1635	0.1165	0.0153	0.0004	0.0064	-0.0221
04:2001	0.5622	0.1604	0.1583	0.1190	0.0103	-0.0070	-0.0101	0.0068
05:2001	0.5514	0.1598	0.1623	0.1264	0.0162	0.0074	-0.0002	-0.0234
06:2001	0.5413	0.1783	0.1605	0.1199	0.0320	-0.0029	-0.0028	-0.0263
07:2001	0.5423	0.1629	0.1849	0.1099	-0.0138	0.0146	-0.0067	0.0059
08:2001	0.5997	0.1477	0.1200	0.1326	0.0217	0.0102	0.0057	-0.0376
09:2001	0.5749	0.1694	0.1673	0.0883	0.0353	0.0138	0.0089	-0.0581
10:2001	0.5741	0.1677	0.1682	0.0899	0.0595	0.0125	0.0139	-0.0860
11:2001	0.5789	0.1667	0.1619	0.0925	0.0581	-0.0313	0.0120	-0.0388
12:2001	0.5523	0.1686	0.1603	0.1188	0.0060	-0.0013	0.0052	-0.0100
01:2002	0.5705	0.1626	0.1610	0.1058	0.0049	-0.0061	0.0086	-0.0074
02:2002	0.5625	0.1592	0.1731	0.1053	0.0066	-0.0161	0.0102	-0.0007
03:2002	0.5452	0.1566	0.1756	0.1227	-0.0070	-0.0113	0.0121	0.0062
04:2002	0.5743	0.1604	0.1616	0.1037	0.0120	0.0000	0.0032	-0.0153
05:2002	0.5294	0.1527	0.1828	0.1351	-0.0220	-0.0071	0.0205	0.0086
06:2002	0.5296	0.1488	0.1650	0.1566	-0.0117	-0.0295	0.0045	0.0368
07:2002	0.5350	0.1596	0.1968	0.1087	-0.0073	-0.0033	0.0119	-0.0012
08:2002	0.6354	0.1355	0.1371	0.0920	0.0358	-0.0122	0.0170	-0.0406
09:2002	0.5658	0.1636	0.1735	0.0971	-0.0091	-0.0059	0.0062	0.0088
10:2002	0.5647	0.1553	0.1769	0.1030	-0.0094	-0.0124	0.0087	0.0131
11:2002	0.5824	0.1479	0.1723	0.0973	0.0035	-0.0188	0.0104	0.0049
12:2002	0.5438	0.1650	0.1816	0.1096	-0.0084	-0.0037	0.0213	-0.0092

Import index prices for Germany, France, Italy and USA (by month, 1989-2002)

Year	**Levels**				**Seasonal differences**			
	Germany	France	Italy	USA	Germany	France	Italy	USA
01:1989	3.1000	3.7397	3.1300	5.8430	-0.1786	0.3922	0.0646	0.0200
02:1989	2.9706	5.1298	3.2427	5.6609	0.0614	1.8381	0.0886	0.0372
03:1989	3.5343	3.1925	3.2600	5.5595	0.0131	-0.4041	-0.0403	0.3136
04:1989	3.2443	3.3248	3.0978	6.9505	-0.2323	-0.1448	0.1439	1.1967
05:1989	3.1789	3.4682	3.1059	6.3173	-0.2071	0.0615	0.1861	0.4955
06:1989	3.5500	3.7081	3.0251	6.4501	0.3143	0.2960	0.0557	0.1706
07:1989	3.5713	3.6331	3.2408	5.5813	0.4154	0.3869	0.0216	-0.0888
08:1989	3.4756	3.3255	3.3968	6.5109	0.2565	-0.1703	0.0895	1.2313
09:1989	3.4340	3.4756	3.2537	6.1678	0.2870	0.2282	0.1531	0.8901
10:1989	3.2172	3.2402	3.0979	6.4674	0.0449	-0.0247	0.1120	0.4562
11:1989	3.2633	3.3882	3.1213	7.0961	0.1364	-0.0932	0.0454	1.3845
12:1989	3.1661	4.0273	3.2993	6.1368	0.1608	0.6333	0.0623	0.0170
01:1990	3.0560	3.3875	3.0828	6.3876	-0.0439	-0.3522	-0.0472	0.5446
02:1990	3.2295	5.2583	3.1431	6.2306	0.2589	0.1285	-0.0996	0.5697
03:1990	3.1372	3.4717	3.2991	6.7496	-0.3971	0.2791	0.0391	1.1901
04:1990	3.0240	3.4406	3.1599	6.5352	-0.2203	0.1158	0.0621	-0.4153
05:1990	3.3659	3.3630	3.0951	7.3676	0.1870	-0.1052	-0.0108	1.0503
06:1990	3.1763	3.5571	3.2742	6.6033	-0.3737	-0.1509	0.2491	0.1532
07:1990	3.2317	3.4193	3.2911	6.0969	-0.3397	-0.2138	0.0502	0.5155
08:1990	3.6242	3.3739	3.4422	5.6423	0.1485	0.0484	0.0453	-0.8687
09:1990	3.5248	3.2914	3.1107	6.1983	0.0908	-0.1841	-0.1430	0.0305
10:1990	3.5593	3.4308	3.0802	5.8972	0.3421	0.1906	-0.0177	-0.5702
11:1990	3.2659	3.4090	3.1134	7.2097	0.0025	0.0208	-0.0079	0.1135
12:1990	3.2978	3.8182	3.3323	5.8119	0.1317	-0.2091	0.0331	-0.3248
01:1991	3.5297	3.5041	3.1604	5.2022	0.4736	0.1166	0.0776	-1.1854
02:1991	3.3401	5.4924	3.3163	6.3191	0.1106	0.2341	0.1733	0.0885
03:1991	3.2711	3.5806	3.2813	6.3254	0.1339	0.1089	-0.0178	-0.4242
04:1991	3.1577	3.5512	3.0685	6.3316	0.1336	0.1106	-0.0914	-0.2035
05:1991	3.0885	3.5496	2.9965	6.4381	-0.2774	0.1866	-0.0986	-0.9296
06:1991	3.0658	3.5686	3.1609	6.3731	-0.1105	0.0115	-0.1133	-0.2303
07:1991	3.0750	3.3653	3.4318	5.9459	-0.1566	-0.0540	0.1407	-0.1510
08:1991	3.5071	3.2970	3.3917	6.3047	-0.1170	-0.0769	-0.0504	0.6625
09:1991	3.2019	3.7123	3.3293	6.1931	-0.3229	0.4208	0.2186	-0.0052
10:1991	3.1758	3.3507	3.2379	6.0607	-0.3835	-0.0801	0.1577	0.1634
11:1991	3.5828	3.3844	3.3583	6.2860	0.3169	-0.0246	0.2449	-0.9237
12:1991	3.6688	4.1179	3.3425	6.9972	0.3710	0.2997	0.0102	1.1852
01:1992	3.3836	3.2839	3.4209	5.4102	-0.1460	-0.2202	0.2605	0.2080
02:1992	3.2814	3.4941	3.3865	5.9914	-0.0587	-1.9983	0.0701	-0.3277
03:1992	3.3381	3.6785	3.4911	6.6364	0.0670	0.0979	0.2098	0.3111
04:1992	3.4101	3.7026	3.4427	6.2148	0.2524	0.1514	0.3742	-0.1169
05:1992	3.0987	4.3366	3.2422	5.7978	0.0101	0.7870	0.2457	-0.6403
06:1992	3.2785	3.8396	3.2974	6.3718	0.2127	0.2709	0.1366	-0.0013
07:1992	3.1880	3.3216	3.4991	5.2135	0.1129	-0.0438	0.0673	-0.7324

Year	Levels				Seasonal differences			
	Germany	France	Italy	USA	Germany	France	Italy	USA
08:1992	3.0346	3.2699	3.4816	6.3835	-0.4725	-0.0270	0.0899	0.0787
09:1992	3.0692	3.4758	3.3298	5.6691	-0.1327	-0.2365	0.0005	-0.5240
10:1992	3.0694	3.2758	3.2831	7.4463	-0.1063	-0.0749	0.0451	1.3857
11:1992	2.9863	3.6367	3.3651	6.7341	-0.5964	0.2522	0.0068	0.4481
12:1992	3.0627	3.4665	3.3920	5.9201	-0.6062	-0.6514	0.0495	-1.0771
01:1993	3.2130	3.5038	3.2905	6.4059	-0.1706	0.2199	-0.1304	0.9957
02:1993	3.3200	3.4340	3.3332	6.1754	0.0386	-0.0601	-0.0533	0.1841
03:1993	3.5317	3.7977	3.3476	5.5651	0.1936	0.1191	-0.1435	-1.0713
04:1993	3.3936	3.4393	3.2371	6.7823	-0.0164	-0.2633	-0.2056	0.5675
05:1993	3.1795	3.3308	3.1472	5.3687	0.0808	-1.0058	-0.0950	-0.4291
06:1993	3.1873	4.0283	3.3102	6.0235	-0.0912	0.1888	0.0128	-0.3483
07:1993	3.1700	3.3581	3.4085	5.7018	-0.0180	0.0365	-0.0907	0.4883
08:1993	3.1314	3.4226	3.6329	5.6680	0.0968	0.1527	0.1512	-0.7155
09:1993	3.2168	3.7413	3.2837	7.2509	0.1476	0.2655	-0.0461	1.5818
10:1993	3.0553	3.5473	3.2565	6.5102	-0.0141	0.2715	-0.0265	-0.9361
11:1993	2.9876	3.4578	3.3985	6.3570	0.0013	-0.1789	0.0334	-0.3771
12:1993	2.9758	3.7502	3.4498	6.4224	-0.0869	0.2836	0.0578	0.5023
01:1994	2.9320	3.4602	3.2370	5.7772	-0.2810	-0.0436	-0.0535	-0.6287
02:1994	3.1367	3.4685	3.2617	5.8841	-0.1832	0.0345	-0.0715	-0.2914
03:1994	3.2605	3.5994	3.3002	6.3537	-0.2712	-0.1983	-0.0474	0.7886
04:1994	3.0018	3.4189	3.2116	6.3455	-0.3919	-0.0204	-0.0255	-0.4368
05:1994	3.4884	3.3878	3.0533	6.2016	0.3089	0.0569	-0.0939	0.8329
06:1994	3.4818	3.3079	3.1104	5.6898	0.2945	-0.7205	-0.1998	-0.3338
07:1994	3.3113	3.2399	3.2460	6.3436	0.1414	-0.1181	-0.1624	0.6417
08:1994	3.4071	3.2657	3.4404	5.9136	0.2757	-0.1569	-0.1925	0.2456
09:1994	3.2426	3.2679	3.1229	5.6431	0.0258	-0.4733	-0.1608	-1.6078
10:1994	3.2169	3.3920	3.2519	5.6085	0.1616	-0.1553	-0.0047	-0.9018
11:1994	3.0025	3.2124	2.9608	5.4414	0.0149	-0.2454	-0.4378	-0.9156
12:1994	3.1597	3.5758	3.3074	5.8317	0.1839	-0.1743	-0.1424	-0.5907
01:1995	3.1653	3.2142	2.9897	5.4945	0.2333	-0.2460	-0.2473	-0.2826
02:1995	3.0681	3.1514	3.1394	5.0010	-0.0686	-0.3171	-0.1223	-0.8831
03:1995	2.9667	3.3033	3.0411	5.4806	-0.2937	-0.2960	-0.2592	-0.8731
04:1995	2.9674	3.6206	2.8794	6.5184	-0.0344	0.2017	-0.3322	0.1729
05:1995	3.6600	3.2796	3.0380	6.2127	0.1716	-0.1082	-0.0153	0.0111
06:1995	3.1678	3.4718	3.2080	6.4202	-0.3140	0.1640	0.0976	0.7304
07:1995	3.3805	3.2425	3.2231	5.0446	0.0692	0.0025	-0.0229	-1.2989
08:1995	3.0629	3.1782	3.3764	5.5055	-0.3442	-0.0875	-0.0640	-0.4081
09:1995	3.5305	3.5424	3.0623	6.1757	0.2879	0.2744	-0.0606	0.5326
10:1995	3.6762	3.4996	3.0418	5.9968	0.4593	0.1076	-0.2101	0.3883
11:1995	3.5203	3.6186	3.0550	5.9586	0.5178	0.4061	0.0942	0.5172
12:1995	3.3699	4.1710	3.2261	6.2761	0.2102	0.5951	-0.0813	0.4444
01:1996	3.2626	3.7266	3.2206	5.7392	0.0973	0.5124	0.2309	0.2446
02:1996	3.4541	3.5562	3.1860	6.4465	0.3860	0.4047	0.0467	1.4455

Year	Levels				Seasonal differences			
	Germany	France	Italy	USA	Germany	France	Italy	USA
03:1996	3.1791	3.7308	3.2302	6.1534	0.2123	0.4275	0.1891	0.6728
04:1996	3.3272	3.8088	3.3041	7.2135	0.3599	0.1883	0.4247	0.6950
05:1996	3.2741	3.7814	3.2103	5.6248	-0.3859	0.5018	0.1722	-0.5879
06:1996	3.1395	3.2450	3.2648	6.5874	-0.0283	-0.2269	0.0568	0.1672
07:1996	3.1415	3.8890	3.3449	5.7560	-0.2390	0.6465	0.1218	0.7114
08:1996	3.0800	3.2636	3.2958	5.7947	0.0171	0.0854	-0.0805	0.2892
09:1996	3.5245	3.2452	3.2730	6.1909	-0.0060	-0.2971	0.2107	0.0153
10:1996	3.3460	3.3675	3.3299	6.4415	-0.3303	-0.1321	0.2881	0.4447
11:1996	3.2085	3.6432	3.3592	6.0319	-0.3118	0.0246	0.3043	0.0733
12:1996	3.5462	4.0712	3.4726	6.1953	0.1763	-0.0998	0.2465	-0.0809
01:1997	3.5892	3.6942	3.5657	5.5013	0.3265	-0.0324	0.3451	-0.2378
02:1997	3.3406	3.5583	3.5835	6.5415	-0.1135	0.0021	0.3975	0.0950
03:1997	3.3463	3.3180	3.5167	6.9059	0.1672	-0.4128	0.2865	0.7525
04:1997	3.1481	3.3752	3.4141	7.2504	-0.1791	-0.4337	0.1100	0.0369
05:1997	3.2711	3.6491	3.2840	7.3091	-0.0030	-0.1323	0.0737	1.6843
06:1997	3.1259	3.7942	3.3244	6.5290	-0.0137	0.5493	0.0596	-0.0584
07:1997	2.9443	4.1780	3.4539	6.4295	-0.1972	0.2890	0.1090	0.6735
08:1997	3.2000	3.4728	3.5099	6.8450	0.1199	0.2093	0.2140	1.0503
09:1997	3.0018	3.3788	3.3275	6.9674	-0.5226	0.1335	0.0545	0.7764
10:1997	3.0382	3.3269	3.2761	6.5334	-0.3078	-0.0406	-0.0538	0.0919
11:1997	3.3221	3.4747	3.3626	7.7116	0.1136	-0.1685	0.0034	1.6797
12:1997	3.3308	3.7131	3.3826	6.3995	-0.2154	-0.3581	-0.0900	0.2042
01:1998	3.1856	3.3574	3.4316	6.0544	-0.4035	-0.3368	-0.1341	0.5531
02:1998	3.4606	3.4322	3.3622	5.6479	0.1200	-0.1261	-0.2213	-0.8936
03:1998	3.3223	3.3251	3.3892	5.9457	-0.0239	0.0070	-0.1276	-0.9602
04:1998	3.2884	3.9430	3.2929	6.9475	0.1402	0.5679	-0.1212	-0.3028
05:1998	3.1583	3.4289	3.2488	7.4350	-0.1128	-0.2201	-0.0351	0.1260
06:1998	3.2701	3.4231	3.4691	6.0834	0.1442	-0.3712	0.1447	-0.4455
07:1998	3.2006	3.4685	3.4736	5.7331	0.2563	-0.7095	0.0197	-0.6964
08:1998	3.2223	3.3013	3.5416	5.8719	0.0223	-0.1716	0.0317	-0.9731
09:1998	3.0886	3.9739	3.3267	6.1058	0.0868	0.5951	-0.0008	-0.8615
10:1998	3.1497	3.4901	3.2739	6.5675	0.1116	0.1632	-0.0022	0.0341
11:1998	3.0239	4.2946	3.3913	6.4283	-0.2983	0.8199	0.0287	-1.2832
12:1998	3.0281	4.1986	3.4278	6.6947	-0.3027	0.4855	0.0453	0.2953
01:1999	2.9996	3.2656	3.2876	5.7348	-0.1861	-0.0917	-0.1441	-0.3196
02:1999	3.4725	4.4330	3.4573	6.1844	0.0119	1.0008	0.0951	0.5365
03:1999	3.1956	4.4280	3.4267	5.9695	-0.1267	1.1029	0.0376	0.0238
04:1999	3.2359	3.3995	3.3140	6.9605	-0.0525	-0.5435	0.0211	0.0129
05:1999	3.5453	4.1146	3.2916	5.9777	0.3870	0.6857	0.0427	-1.4573
06:1999	3.5744	4.4544	3.4758	6.0508	0.3043	1.0313	0.0068	-0.0326
07:1999	3.4720	4.2423	3.3072	6.0710	0.2713	0.7738	-0.1664	0.3378
08:1999	3.3734	4.4488	3.3521	6.0160	0.1511	1.1475	-0.1895	0.1441
09:1999	3.2664	4.1190	3.4795	6.5089	0.1777	0.1451	0.1528	0.4031

Year	Levels				Seasonal differences			
	Germany	France	Italy	USA	Germany	France	Italy	USA
10:1999	3.5167	4.2295	3.2983	6.2138	0.3670	0.7394	0.0244	-0.3537
11:1999	3.1126	3.5194	3.4679	6.8218	0.0887	-0.7752	0.0766	0.3935
12:1999	3.1976	4.7640	3.5923	6.4109	0.1694	0.5654	0.1645	-0.2839
01:2000	3.1903	3.5606	3.4597	6.2708	0.1907	0.2950	0.1721	0.5360
02:2000	3.1483	3.5463	3.5488	6.2399	-0.3241	-0.8867	0.0914	0.0555
03:2000	3.0463	3.9817	3.3633	7.5812	-0.1493	-0.4463	-0.0635	1.6117
04:2000	2.9979	3.5217	3.4429	5.8504	-0.2380	0.1222	0.1289	-1.1101
05:2000	3.0333	3.4441	3.3251	6.5507	-0.5120	-0.6705	0.0335	0.5730
06:2000	3.1639	4.2792	3.3631	6.3792	-0.4106	-0.1752	-0.1128	0.3283
07:2000	3.5889	3.4717	3.5844	6.2119	0.1170	-0.7706	0.2773	0.1410
08:2000	3.5446	3.4674	3.4966	6.8661	0.1712	-0.9814	0.1444	0.8500
09:2000	3.6136	3.5338	3.4085	8.1616	0.3472	-0.5853	-0.0710	1.6527
10:2000	3.4617	3.7423	3.5065	7.6396	-0.0551	-0.4873	0.2081	1.4259
11:2000	3.3339	4.5105	3.5943	6.7192	0.2213	0.9911	0.1264	-0.1026
12:2000	3.2511	4.0392	3.6217	6.1728	0.0535	-0.7249	0.0294	-0.2381
01:2001	3.1938	3.6394	3.4963	6.3619	0.0035	0.0788	0.0366	0.0911
02:2001	3.1898	3.9710	3.4947	6.4879	0.0414	0.4248	-0.0540	0.2480
03:2001	3.1286	3.8871	3.7034	7.3510	0.0823	-0.0946	0.3401	-0.2302
04:2001	3.1456	3.8665	3.5969	6.0972	0.1477	0.3448	0.1539	0.2468
05:2001	3.0588	3.9438	3.5613	7.0191	0.0255	0.4997	0.2362	0.4684
06:2001	3.0333	4.1600	3.5794	7.3665	-0.1305	-0.1192	0.2163	0.9873
07:2001	3.0294	3.8714	3.7461	6.2442	-0.5596	0.3997	0.1617	0.0323
08:2001	3.1854	3.8867	3.7001	7.3625	-0.3592	0.4193	0.2035	0.4965
09:2001	3.5079	3.8627	3.5048	5.7526	-0.1057	0.3289	0.0963	-2.4091
10:2001	3.2110	3.6843	3.5332	5.8443	-0.2506	-0.0579	0.0267	-1.7954
11:2001	3.9651	4.0467	3.5531	5.9768	0.6312	-0.4638	-0.0412	-0.7424
12:2001	3.6251	4.3093	3.6340	6.8885	0.3740	0.2701	0.0123	0.7157
01:2002	3.6294	3.6976	3.5351	5.9551	0.4356	0.0583	0.0388	-0.4068
02:2002	3.4692	3.7012	3.5173	6.4065	0.2794	-0.2699	0.0225	-0.0814
03:2002	3.3253	3.7965	3.8541	7.8319	0.1966	-0.0906	0.1507	0.4809
04:2002	3.3203	3.7144	3.4474	6.3610	0.1747	-0.1521	-0.1494	0.2638
05:2002	3.2448	3.6922	3.6030	7.7594	0.1861	-0.2517	0.0417	0.7403
06:2002	3.2256	3.7327	3.6143	8.5515	0.1922	-0.4273	0.0349	1.1850
07:2002	3.3146	3.9628	3.8339	6.8334	0.2852	0.0914	0.0878	0.5892
08:2002	3.2594	3.6249	4.0430	5.9423	0.0740	-0.2618	0.3429	-1.4202
09:2002	3.1753	3.5647	3.6973	7.1220	-0.3326	-0.2980	0.1925	1.3695
10:2002	3.1733	3.6231	3.7712	7.4512	-0.0378	-0.0613	0.2380	1.6070
11:2002	3.6060	3.6659	3.9415	6.1618	-0.3591	-0.3809	0.3885	0.1849
12:2002	3.2389	4.2915	4.0897	6.9059	-0.3862	-0.0178	0.4557	0.0174

Real expenditure term (by month, 1989-2002)

Month/ Year	Levels	Seasonal differences	Month/ Year	Levels	Seasonal differences
01:1989	5.7058	0.0986	08:1992	5.6872	0.3492
02:1989	5.4770	-0.3344	09:1992	5.2231	0.2671
03:1989	5.4631	0.0371	10:1992	5.0483	-0.0060
04:1989	5.3417	-0.0055	11:1992	4.7226	0.2688
05:1989	5.4523	0.0560	12:1992	5.0794	0.5332
06:1989	5.1941	-0.1900	01:1993	4.9156	-0.1339
07:1989	5.2347	-0.2187	02:1993	5.2366	-0.0392
08:1989	5.2093	-0.3188	03:1993	5.1915	0.0107
09:1989	5.2618	-0.3290	04:1993	4.0301	0.0437
10:1989	5.5458	-0.0896	05:1993	4.8651	0.1355
11:1989	5.3598	-0.1828	06:1993	4.8757	0.1075
12:1989	5.2828	-0.1818	07:1993	5.5931	-0.0758
01:1990	5.8145	0.1117	08:1993	4.9500	-0.0223
02:1990	5.2468	-0.1374	09:1993	4.7725	-0.3410
03:1990	5.5696	0.1207	10:1993	4.7077	-0.0133
04:1990	5.5527	0.2588	11:1993	5.0889	0.0559
05:1990	5.1389	-0.2353	12:1993	4.9577	-0.0459
06:1990	5.3641	0.1233	01:1994	5.5975	0.2390
07:1990	5.8036	0.1526	02:1994	5.0939	0.1287
08:1990	4.9685	0.0496	03:1994	4.7049	0.1359
09:1990	5.4401	0.0346	04:1994	4.9794	0.2773
10:1990	5.4701	-0.1154	05:1994	5.0684	-0.2124
11:1990	5.5160	0.0088	06:1994	5.5902	0.0687
12:1990	5.3200	-0.0333	07:1994	5.2994	-0.0928
01:1991	5.3643	-0.1753	08:1994	4.6150	-0.1322
02:1991	5.1557	-0.2225	09:1994	5.3045	0.3014
03:1991	5.4380	-0.1802	10:1994	5.4012	0.0738
04:1991	5.3280	-0.2073	11:1994	5.4474	0.2654
05:1991	5.5220	0.1934	12:1994	5.2455	0.0704
06:1991	5.3441	-0.0085	01:1995	5.7406	0.0498
07:1991	5.6351	0.0971	02:1995	5.8239	0.3033
08:1991	5.2732	-0.1273	03:1995	5.7989	0.4179
09:1991	5.1955	-0.0005	04:1995	5.4440	0.0883
10:1991	5.5733	0.0823	05:1995	5.4454	-0.0180
11:1991	5.1463	-0.2450	06:1995	5.1882	0.0250
12:1991	4.6069	-0.4191	07:1995	5.7077	0.1658
01:1992	5.5594	0.0086	08:1995	5.7717	0.3528
02:1992	5.7134	0.3919	09:1995	5.2836	-0.2755
03:1992	5.2961	-0.0729	10:1995	5.3140	-0.2769
04:1992	5.0638	-0.2092	11:1995	5.4131	-0.4403
05:1992	5.3015	-0.0166	12:1995	4.7900	-0.3399
06:1992	5.1347	-0.0949	01:1996	4.9845	-0.1369
07:1992	5.6575	0.0150	02:1996	5.1087	-0.4791

Month/ Year	Levels	Seasonal differences	Month/ Year	Levels	Seasonal differences
03:1996	5.3523	-0.4115	09:1999	5.1547	-0.1448
04:1996	4.9234	-0.5012	10:1999	4.9835	-0.1882
05:1996	5.2060	0.1023	11:1999	5.5051	-0.0325
06:1996	5.1987	-0.0351	12:1999	5.0870	-0.1282
07:1996	5.3156	-0.0946	01:2000	5.1093	-0.2349
08:1996	5.5239	-0.1537	02:2000	5.6275	0.3516
09:1996	5.1750	-0.0216	03:2000	5.1651	-0.0777
10:1996	5.2754	0.0902	04:2000	4.7069	0.2713
11:1996	5.2587	0.1237	05:2000	5.2922	0.3196
12:1996	4.7133	-0.1383	06:2000	4.9744	0.1398
01:1997	4.6945	-0.1988	07:2000	5.2247	0.0273
02:1997	5.3600	-0.0646	08:2000	5.1064	-0.1618
03:1997	4.7414	-0.2126	09:2000	4.7201	-0.4271
04:1997	4.9585	0.1561	10:2000	4.7657	-0.2740
05:1997	4.8612	-0.2860	11:2000	4.9230	-0.3134
06:1997	5.0558	-0.1001	12:2000	4.9284	0.1242
07:1997	5.3355	-0.0472	01:2001	5.5442	0.0944
08:1997	5.1185	-0.4317	02:2001	5.1520	-0.0708
09:1997	5.1030	0.1749	03:2001	5.0282	0.0563
10:1997	5.4001	0.2267	04:2001	5.2479	-0.1253
11:1997	4.7453	-0.3187	05:2001	5.1003	-0.1074
12:1997	5.0142	0.2052	06:2001	5.2539	0.0552
01:1998	4.8611	0.1597	07:2001	5.6481	0.2418
02:1998	5.5122	0.1589	08:2001	4.7286	0.1914
03:1998	4.1850	0.2388	09:2001	4.4149	0.4301
04:1998	5.0434	0.0053	10:2001	5.3964	0.6262
05:1998	5.1296	0.0769	11:2001	4.7931	-0.1184
06:1998	5.3040	0.1434	12:2001	5.1485	-0.3485
07:1998	5.3097	0.1551	01:2002	5.3078	-0.2264
08:1998	5.6057	0.3976	02:2002	5.4735	-0.1339
09:1998	5.4888	0.0402	03:2002	5.2528	-0.2685
10:1998	5.4922	-0.1557	04:2002	5.5295	-0.0419
11:1998	5.6307	0.3990	05:2002	5.0797	-0.2658
12:1998	4.4536	0.0760	06:2002	4.9623	-0.3785
01:1999	5.5691	0.2572	07:2002	5.3393	-0.2659
02:1999	5.3967	-0.2230	08:2002	5.5095	0.1788
03:1999	5.2889	-0.0506	09:2002	5.4259	0.0671
04:1999	5.0891	0.1074	10:2002	5.6148	-0.2002
05:1999	4.9644	0.0097	11:2002	5.3448	0.1793
06:1999	5.1455	-0.3105	12:2002	5.2843	0.1712
07:1999	5.1473	-0.3140			
08:1999	5.1815	-0.2159			

Bibliography

Amemiya, T. (1985). *Advanced Econometrics*. Harvard University Press, Cambridge, Massachussetts.

Anderson, G. J. and Blundell, R. W. (1982). Estimation and hypothesis testing in a dynamic equation system. *Economerica*, 50:1559–1571.

Anderson, G. J. and Blundell, R. W. (1983). Testing restrictions in a flexible dynamic demand system: an application to consumers' expenditure in canada. *Review of Economic Studies*, 50:397–410.

Anderson, G. J. and Blundell, R. W. (1984). Consumer non-durables in the uk: a dynamic demand system. *Economic Journal*, 94:35–44.

Anderson, T. W. (1958). *An Introduction to Multivariate Statistical Analysis*. Wiley, New York.

Assarsson, B. (1996). *The Almost Ideal Demand System in Error Correction Form*, volume 34 of *Advanced Studies in Theoretical and Applied Econometrics*, chapter 7, pages 195–204. Kluwer Academic Publishers, Dordrech, Boston, London.

Barten, A. P. (1969). Maximum likelihood estimation of a complete system of demand equations. *European Economic Review*, 1:7–73.

Berndt, E. and Savin, N. E. (1975). Estimation and hypothesis testing in singular equation systems with autoregressive disturbance. *Econometrica*, 43:937–957.

Bewley, R. (1986). *Allocation Models: Specification, Estimation and Applications.* Ballinger Publishing Company, Cambridge.

Blackorby, C., Primont, D., and Russel, R. R. (1978). *Duality, Separability, and Functional Structure: Theory and Economic Applications.* A Series of Volumes in Dynamic Economics: Theory and Applications. North-Holland, New York.

Burgess, D. F. (1974). A cost minimization approach to import demand equations. *The Review of Economics and Statistics*, 56:225–234.

Davidson, J., Hendry, D., Srba, F., and Yeo, S. (1978). Econometric modelling of the aggregate time-series relationship between consumers' expenditure and income in the uk. *Economic Journal*, 88:661–692.

Davidson, R. and MacKinnon, J. G. (1993). *Estimation and Inference in Econometrics.* Oxford University Press, New York.

Deaton, A. S. and Muellbauer, J. (1980a). An almost ideal demand system. *American Economic Review*, 70:312–326.

Deaton, A. S. and Muellbauer, J. (1980b). *Economics and Consumer Behavior*. Cambridge University Press, London.

Deschamps, P. J. (1993). Joint test for regularity and autocorrelation in allocation systems. *Journal of Applied Econometrics*, 8(2):195–211.

Deschamps, P. J. (1996). Monte carlo methodology for lm and lr autocorrelation tests in multivariate regression. *Annales d'Economie et de Statistique*, (43):149–169.

Deschamps, P. J. (1998). Full maximum likelihood estimation of dynamic demand models. *Journal of Econometrics*, 82:335–359.

Deschamps, P. J. (2003). Time-varying intercepts and equilibrium analysis: an extension of the dynamic almost ideal demand model. *Journal of Applied Econometrics*, 18:209–236.

Diewert, W. and Morrison, C. J. (1986). Export supply and import demand functions: a production theory approach. NBER Working Paper Series 2011, NBER, Cambridge MA.

Diewert, W. E. and Morrison, C. J. (1988). *Export Supply and Import Demand Functions: a Production Theory Approach*, pages 207–229. MIT Press, Cambridge MA.

Edgerton, D. L., Assarsson, B., Hummelmose, A., Luarila, I. P., Rickertsen, K., and Vale, P. H. (1996). *The Econometrics of Demand Systems: with Application to Food Demand in the Nordic Countries*, volume 34 of *Advanced Studies in Theoretical and Applied Econometrics*. Kluwer Academic Publishers, Dordrech, Boston, London.

Green, R. and Alston, J. M. (1990). Elasticities in aids models. *American Journal of Agricultural Economics*, 72:442–445.

Greene, W. H. (1997). *Econometric Analysis*. Prentice Hall, Upper Saddle River, NJ, third edition.

Hamilton, J. D. (1994). *Time Series Analysis*. Princeton University Press, Princeton.

Hendry, D. F. (1995). *Dynamic Econometrics*. Advanced Texts in Econometrics. Oxford University Press, Oxford, New York.

Italianer, A. (1985). A small sample correction for the likelihood ratio test. *Economics Letters*, 19:315–317.

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control*, 12:231–254.

Johansen, S. (1991). Estimation and hypothesis testing of cointegration vectors in gaussian vector autoregressive models. *Econometrica*, 59(6):1551–1580.

Judge, G. G., Griffiths, W. E., Carter-Hill, R., Lütkenpohl, H., and Lee, T.-C. (1985). *The Theory and Practice of Econometrics*. Wiley, New York, second edition.

Kiviet, J. (1986). On the rigour of some misspecification tests for modelling dynamic relationships. *Review of Economic Studies*, 53:241–261.

Kohli, U. R. (1978). A gross national product function and the derived demand for imports and supply of exports. *The Canadian Journal of Economics*, 11:167–182.

Laitinen, K. (1978). Why is demand homogeneity so often rejected? *Economic Letters*, 1:187–191.

Lau, L. J. (1978). A note on the compatibility of a system of difference equations and a time-independent linear equation. *Economic Letters*, 1:243–247.

Magnus, J. R. and Neudecker, H. (1988). *Matrix Differential Calculus with Applications in Statistics and Econometrics*. Wiley, New York.

Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995). *Microeconomic Theory*. Oxford University Press, New York.

Pollak, R. A. and Wales, T. J. (1992). *Demand System Specification and Estimation*. Oxford University Press, New York.

Theil, H. and Clements, K. W. (1987). *Applied Demand Analysis: Results from System-Wide Approaches*. Series on Econometrics and Management Sciences. Ballinger Publishing Company, Cambridge MA.

van Heeswijk, B., Boer, P. D., and Harkema, R. (1993). A dynamic specification of an aids import allocation model. *Review of Empirical Economics*, 18(1):57–73.

Winters, L. A. (1984). Separability and the specification of foreign trade functions. *Journal of International Economics*, 17:239–263.